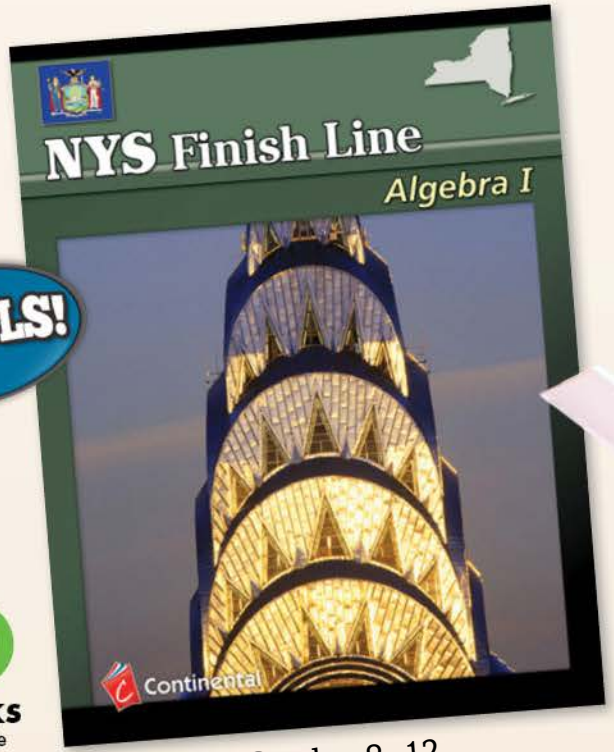
NYS Finish Line Algebra I



For CCLS!

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CONTENTS

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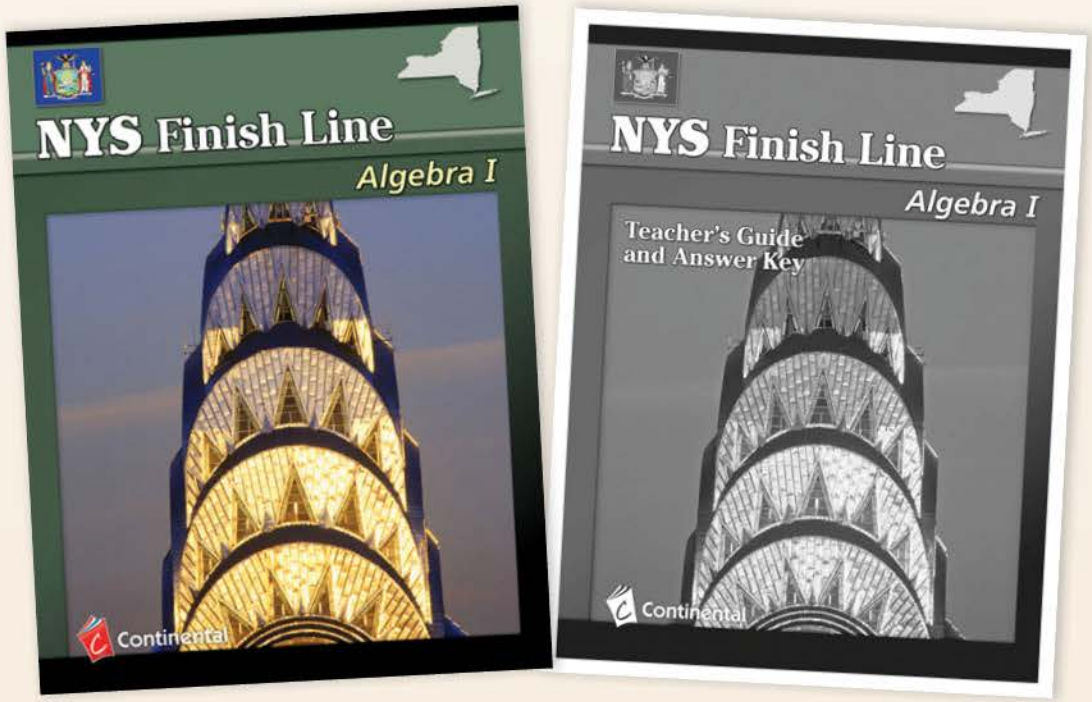
What does the book do?

NYS Finish Line Algebra I provides instruction and practice for the New York Common Core Learning Standards (CCLS) and prepares students for the Regents Algebra I (Common Core) exam. Components include a student workbook and a teacher's guide in print and eBook formats.



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Standards Alignment

The content addresses all focus standards identified for the traditional Algebra I course assessed by the exam. The modules follow the same sequence as those identified for this course in the *NYS Common Core Mathematics Curriculum for Algebra I* as described by EngageNY.org. Each module is broken down into lessons that address one or more standards.

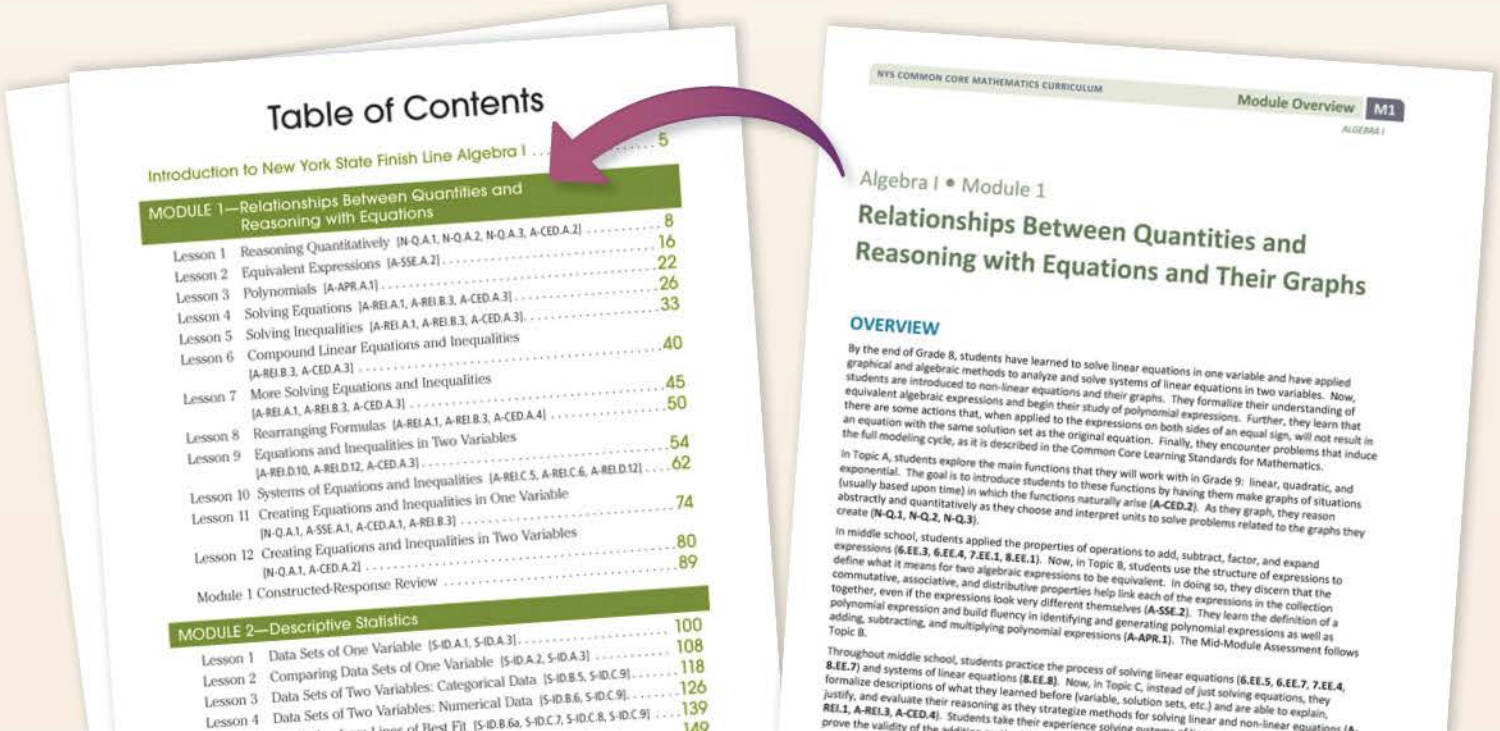


Table of Contents

Algebra I • Module 1

Relationships Between Quantities and Reasoning with Equations and Their Graphs

OVERVIEW

By the end of Grade 8, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students are introduced to non-linear equations and their graphs. They formalize their understanding of equivalent algebraic expressions and begin their study of polynomial expressions. Further, they learn that there are some actions that, when applied to the expressions on both sides of an equal sign, will not result in an equation with the same solution set as the original equation. Finally, they encounter problems that induce the full modeling cycle, as it is described in the Common Core Learning Standards for Mathematics.

In Topic A, students explore the main functions that they will work with in Grade 9: linear, quadratic, and exponential. The goal is to introduce students to these functions by having them make graphs of situations (usually based upon time) in which the functions naturally arise (**A-CED.2**). As they graph, they reason abstractly and quantitatively as they choose and interpret units to solve problems related to the graphs they create (**N-Q.1, N-Q.2, N-Q.3**).

In middle school, students applied the properties of operations to add, subtract, factor, and expand expressions (**6.EE.3, 6.EE.4, 7.EE.1, 8.EE.1**). Now, in Topic B, students use the structure of expressions to define what it means for two algebraic expressions to be equivalent. In doing so, they discern that the commutative, associative, and distributive properties help link each of the expressions in the collection together, even if the expressions look very different themselves (**A-SSE.2**). They learn the definition of a polynomial expression and build fluency in identifying and generating polynomial expressions as well as adding, subtracting, and multiplying polynomial expressions (**A-APR.1**). The Mid-Module Assessment follows Topic B.

Throughout middle school, students practice the process of solving linear equations (**6.EE.5, 6.EE.7, 7.EE.4, 8.EE.7**) and systems of linear equations (**8.EE.8**). Now, in Topic C, instead of just solving equations, they formalize descriptions of what they learned before (variable, solution sets, etc.) and are able to explain, justify, and evaluate their reasoning as they strategize methods for solving linear and non-linear equations (**A-REI.1, A-REI.3, A-CED.4**). Students take their experience solving systems of linear equations (**A-**

prove the validity of the addition

Rigorous Lessons

Rigorous content meets the demands of the standards and exam. Questions range in difficulty, with many Depth of Knowledge (DOK) levels 2 and higher.

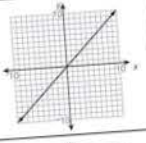
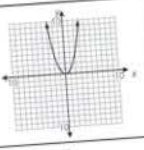
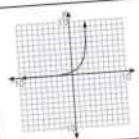
LESSON 2

Choosing Appropriate Functions

F-LE.A.1b, c

A Chart of Parent Functions

Functions can be recognized by key features of their graphs. This chart shows a summary of some function equations, their parent graphs, and their key features.

Parent Function	Graph of Parent Function	Key Features
Linear $f(x) = x$		• Graph is a straight line • Average rates of change per unit interval are constant • Always has one y-intercept for non-vertical lines
Absolute Value (Piecewise) $f(x) = \lvert x \rvert$		• Graph is a V-shape • Has a vertex, or minimum or maximum point • Average rates of change per unit interval are constant for all points on the same side of the vertex • Always has one y-intercept
Quadratic $f(x) = x^2$		• Graph is a U-shape • Has a vertex • Average rates of change per unit interval differ • Always has one y-intercept
Exponential $f(x) = b^x$		• Shape shows growth (as x increases, y increases) or decay (as x increases, y decreases) • Average rates of change per unit interval differ

Try this sample question.

S-1 Corbin started a job at the beginning of the year. He gets paid $40,000 for the year. Corbin expects to earn a 4% pay increase each year he is at this job. Which type of function is best used to predict the amount of pay Corbin can expect to get during the next few years?

A linear function
B absolute value function
C quadratic function
D exponential function

A function that models growth by a constant factor, 4%, best models this situation. The change in pay from one year to the next can be determined by multiplying the previous year's pay by the pay increase rate. Exponential models show this type of growth. Choice D is correct.

Some functions can be identified by analyzing rates of change per unit intervals in a table of values. To analyze rate of change in an x–y table where the x-values increase by 1, compare the change in consecutive y-values.

A **unit interval** is measured by an x increase of 1.

- If the **first differences** in y-values are constant, then the data set could model a linear function. For example:

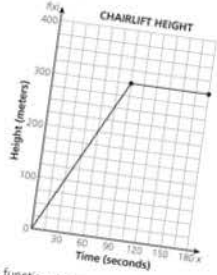
x	y	Difference between y-values
0	3	
1	9	$9 - 3 = \mathbf{6}$
2	15	$15 - 9 = \mathbf{6}$
3	21	$21 - 15 = \mathbf{6}$
4	27	$27 - 21 = \mathbf{6}$

- If the **second differences,** or differences between the differences in y-values, are constant, then the data set could model a quadratic function. For example:

x	y	Difference between y-values	Difference between differences between y-values
0	0		
1	2	$2 - 0 = 2$	
2	8	$8 - 2 = 6$	$6 - 2 = \mathbf{4}$
3	18	$18 - 8 = 10$	

Read this problem. Write your answers.

5 The graph below shows the relationship between the height of a chairlift, in meters, and the time, in seconds, the chairlift is moving, over a 180-second period.

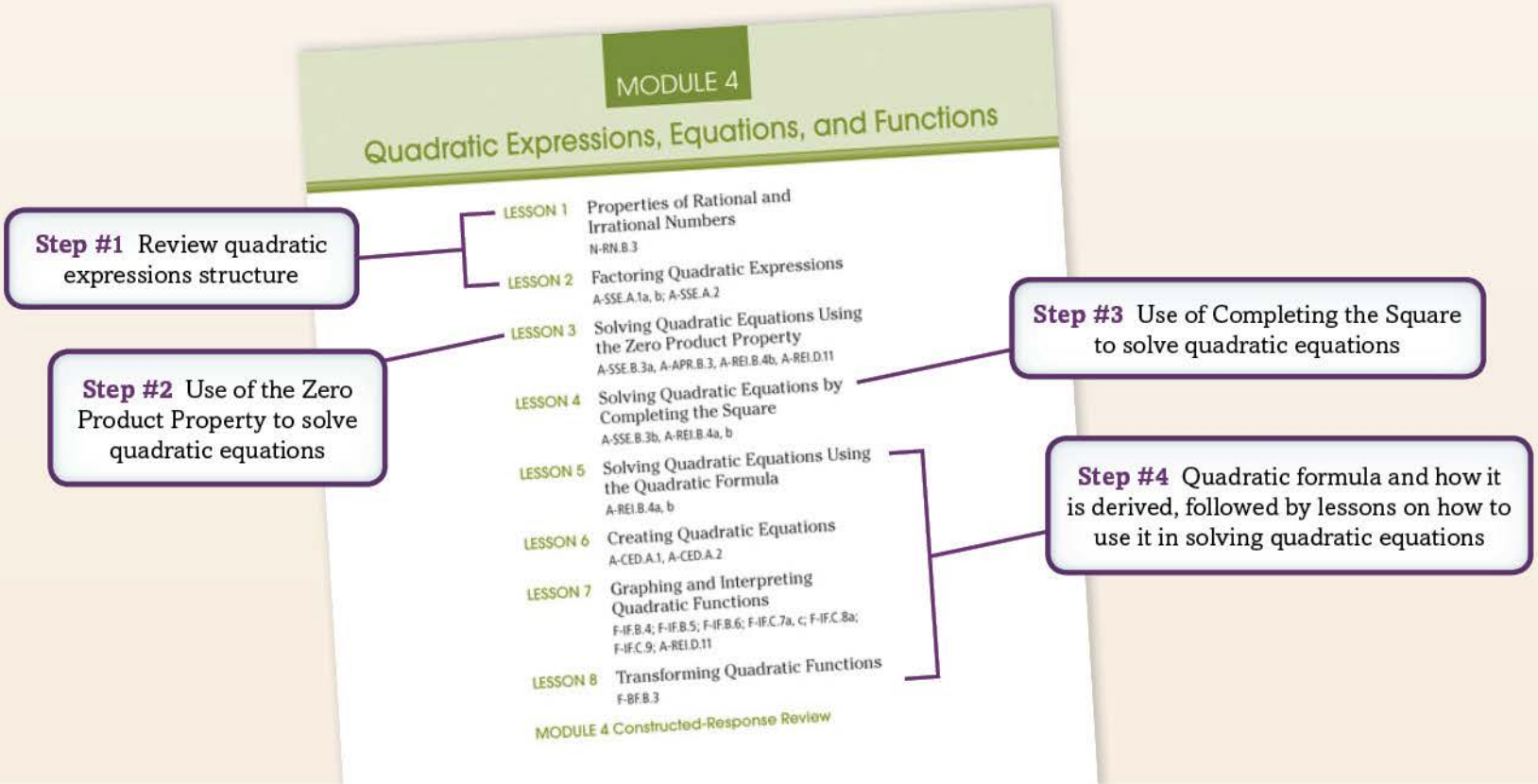


Write a piecewise function to represent the situation modeled by the graph. Show your work.

Student Book 4

Skill Building

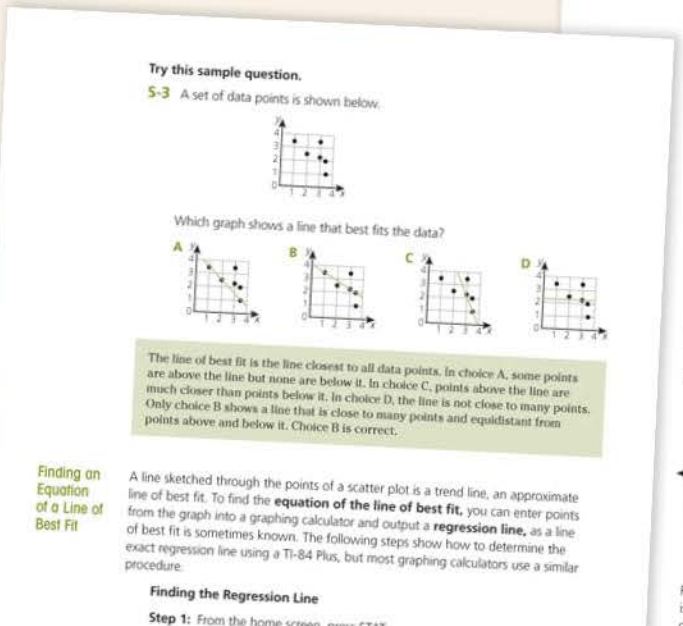
Topics that are often stumbling blocks for students are covered in detail, beginning with a careful explanation of fundamentals and building on those to an understanding of processes.



Learning Support

Features to support student comprehension:

- Guided examples with insight into the topic and procedures for finding the answer
- Sidebar items with key terms and concepts
- Calculator instructions with important procedures
- Charts, tables and graphs, illustrations and diagrams



Try this sample question.

S-3 A set of data points is shown below.

Which graph shows a line that best fits the data?

A

B

C

D

The line of best fit is the line closest to all data points. In choice A, some points are above the line but none are below it. In choice C, points above the line are much closer than points below it. In choice D, the line is not close to many points. Only choice B shows a line that is close to many points and equidistant from points above and below it. Choice B is correct.

Finding an Equation of a Line of Best Fit

A line sketched through the points of a scatter plot is a trend line, an approximate line of best fit. To find the **equation of the line of best fit,** you can enter points from the graph into a graphing calculator and output a **regression line,** as a line of best fit is sometimes known. The following steps show how to determine the exact regression line using a TI-84 Plus, but most graphing calculators use a similar procedure.

Finding the Regression Line

Step 1: From the home screen, press STAT.

$y = 2x + 1$

$y > 2x + 1$

The graph of the inequality $y > 2x + 1$ is exactly the area ***above*** the graph of the line $y = 2x + 1$, as shown in the graph above. To determine which side of the graph is shaded, a test point can be substituted into the original inequality. If that solution is true, the side containing the test point is shaded. Otherwise, the opposite side is shaded. Notice that when the line itself is ***not*** part of the graph, the line drawn is dashed.

Graphs of inequalities with $\geq$ or $\leq$ include the line.

Graphs of inequalities with $>$ or $<$ do ***not*** include the line.

The following examples show the difference in the solution to the above inequality when the inequality sign is changed:

$y \geq 2x + 1$ $y \leq 2x + 1$ $y < 2x + 1$

For an inequality in two variables, a **solution** is an ordered pair if, when each number is substituted into its corresponding variable, the inequality is true. The **solution set**

Solving Equations by Factoring

You can solve a quadratic equation by factoring the quadratic expression and applying the zero product property.

Remember to set quadratic equations equal to 0 before solving.

For example, consider the quadratic equation $x^2 + 6x + 5 = 0$. Factoring $x^2 + 6x + 5 = 0$ gives $(x + 1)(x + 5) = 0$. According to the zero product property, either $x + 1 = 0$ or $x + 5 = 0$. So, $x = -1$ or $x = -5$.

Try this sample question.

S-3 What is the solution to the equation $3z^2 - 2 = 5z$?

A $z = \frac{1}{3}$ or $z = -2$

B $z = -\frac{1}{3}$ or $z = 2$

C $z = \frac{5}{3}$ or $z = -1$

D $z = -\frac{5}{3}$ or $z = 1$

Set the equation equal to 0 by subtracting $5z$ from each side:

$3z^2 - 5z - 2 = 0$

Factor the quadratic expression by finding factors of 3 and -2 that combine to equal -5:

Factors	Combinations of Factors	
1 and 3; 1 and -2	$1 \cdot 1 + 3 \cdot -2 = -5$ ✓	$1 \cdot -2 + 3 \cdot 1 = 1$
-1 and 2	$1 \cdot -1 + 3 \cdot 2 = 5$	$1 \cdot 2 + 3 \cdot -1 = -1$

The factors of -2 that combine with the factors of 3 are 1 and -2. Since $1z$ multiplies 1, these terms are placed in opposite factors. Likewise, since 3 multiplies -2, $3z$ is placed in the opposite factor as -2. So, $3z^2 - 5z - 2 = (3z + 1)(1z - 2)$ or $(3z + 1)(z - 2)$.

$(3z + 1)(z - 2) = 0$

Apply the zero product property by setting each separate factor equal to 0. Then solve for the variable:

$3z + 1 = 0$ or $z - 2 = 0$

$z = -\frac{1}{3}$ or $z = 2$

Choice B is correct.

Keep in mind that some solutions to a quadratic equation may not be appropriate given the context of the problem. For example, suppose you are solving an equation representing the area of a rectangle. Any solution to the equation that results in a side length of the rectangle being negative would not be appropriate since length must be a positive value.

Question Types

- Multiple-choice items and 2-, 4-, and 6-point constructed-response problems test multiple anchors.
- Module reviews can be used as practice tests.

3 A certain substance decays at an exponential rate. The amount of the substance on any given day is 50% of the amount of the substance the previous day. Which function best models the amount of 1,000 grams of the substance remaining after x days?

A $f(x) = 1{,}000(0.50x)$

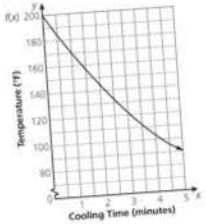
B $f(x) = 1{,}000(0.50^x)$

C $f(x) = 1{,}000.50^x$

D $f(x) = 1{,}000(1.50^x)$

Read the problem. Write your answers.

4 Beatrice recorded the temperature of a cup of hot water as it cooled during an experiment. The graph below shows the temperature, in degrees Fahrenheit, of the cup of hot water x minutes after cooling.



Write a function that models this situation. Show or explain all steps you used to determine the function.

Module 4
Constructed-Response Review

Read each problem. Write your answers.

1 The area of the triangular sail shown below is 110 square feet.

A Write an equation for the area of the sail.

B Solve the equation you wrote in part A to find all values of x. Show or explain how you determined your answer.

C Explain how you know which value of x to use to find the area of the sail.

2 The area of a rectangle, in square units, is defined by the expression $(x + \pi)(x + 5)$. In the expression,

- the value of x is rational, and
- the length of the rectangle is rational.

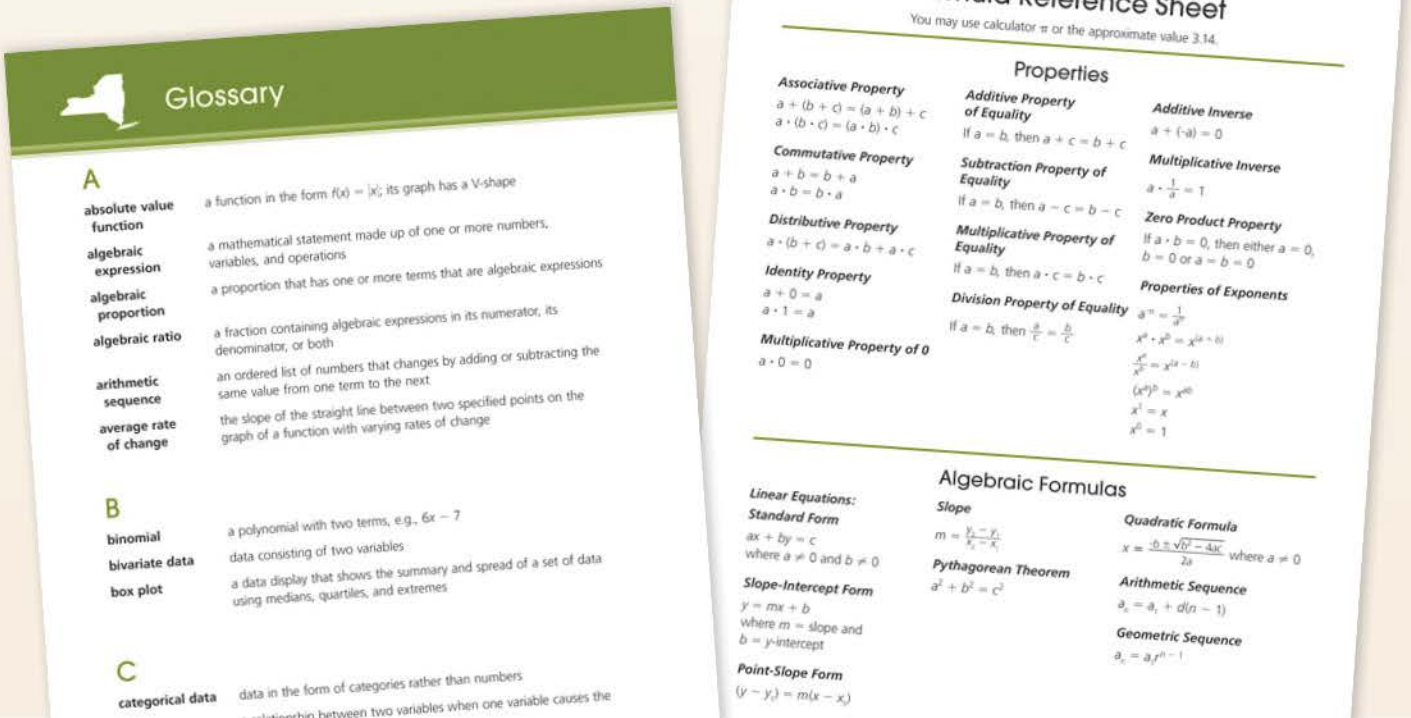
A Write an expression for the width, in units, of the rectangle. Explain how you know it is the width of the rectangle.

B Is the area of this rectangle rational or irrational? Justify your answer.

C The area of a different rectangle is equal to $x^2 - \pi^2$ square units. Write expressions for the length, in units, and the width, in units, of the different rectangle. Explain how you determined each expression.

Additional Resources

- The glossary includes terms that are boldfaced throughout the book and reflect the vocabulary identified by the NYSCC Mathematics Curriculum as important to know.
- A reference sheet includes many of the formulas that appear on the Regents test formula sheet.



Glossary

A

absolute value function — a function in the form $f(x) = |x|$; its graph has a V-shape

algebraic expression — a mathematical statement made up of one or more numbers, variables, and operations

algebraic proportion — a proportion that has one or more terms that are algebraic expressions

algebraic ratio — a fraction containing algebraic expressions in its numerator, its denominator, or both

arithmetic sequence — an ordered list of numbers that changes by adding or subtracting the same value from one term to the next

average rate of change — the slope of the straight line between two specified points on the graph of a function with varying rates of change

B

binomial — a polynomial with two terms, e.g., $6x - 7$

bivariate data — data consisting of two variables

box plot — a data display that shows the summary and spread of a set of data using medians, quartiles, and extremes

C

categorical data — data in the form of categories rather than numbers

Formula Reference Sheet

You may use calculator π or the approximate value 3.14.

Properties

Associative Property
$a + (b + c) = (a + b) + c$
$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

Commutative Property
$a + b = b + a$
$a \cdot b = b \cdot a$

Distributive Property
$a \cdot (b + c) = a \cdot b + a \cdot c$

Identity Property
$a + 0 = a$
$a \cdot 1 = a$

Multiplicative Property of 0
$a \cdot 0 = 0$

Additive Property of Equality
If $a = b$, then $a + c = b + c$

Subtraction Property of Equality
If $a = b$, then $a - c = b - c$

Multiplicative Property of Equality
If $a = b$, then $a \cdot c = b \cdot c$

Division Property of Equality
If $a = b$, then $\frac{a}{c} = \frac{b}{c}$

Additive Inverse
$a + (-a) = 0$

Multiplicative Inverse
$a \cdot \frac{1}{a} = 1$

Zero Product Property
If $a \cdot b = 0$, then either $a = 0$, $b = 0$ or $a = b = 0$

Properties of Exponents
$a^{-n} = \frac{1}{a^n}$
$x^a \cdot x^b = x^{(a+b)}$
$\frac{x^a}{x^b} = x^{(a-b)}$
$(x^a)^b = x^{ab}$
$x^1 = x$
$x^0 = 1$

Algebraic Formulas

Linear Equations: Standard Form
$ax + by = c$
where $a \neq 0$ and $b \neq 0$

Slope-Intercept Form
$y = mx + b$
where m = slope and b = y-intercept

Point-Slope Form
$(y - y_1) = m(x - x_1)$

Slope
$m = \frac{y_2 - y_1}{x_2 - x_1}$

Pythagorean Theorem
$a^2 + b^2 = c^2$

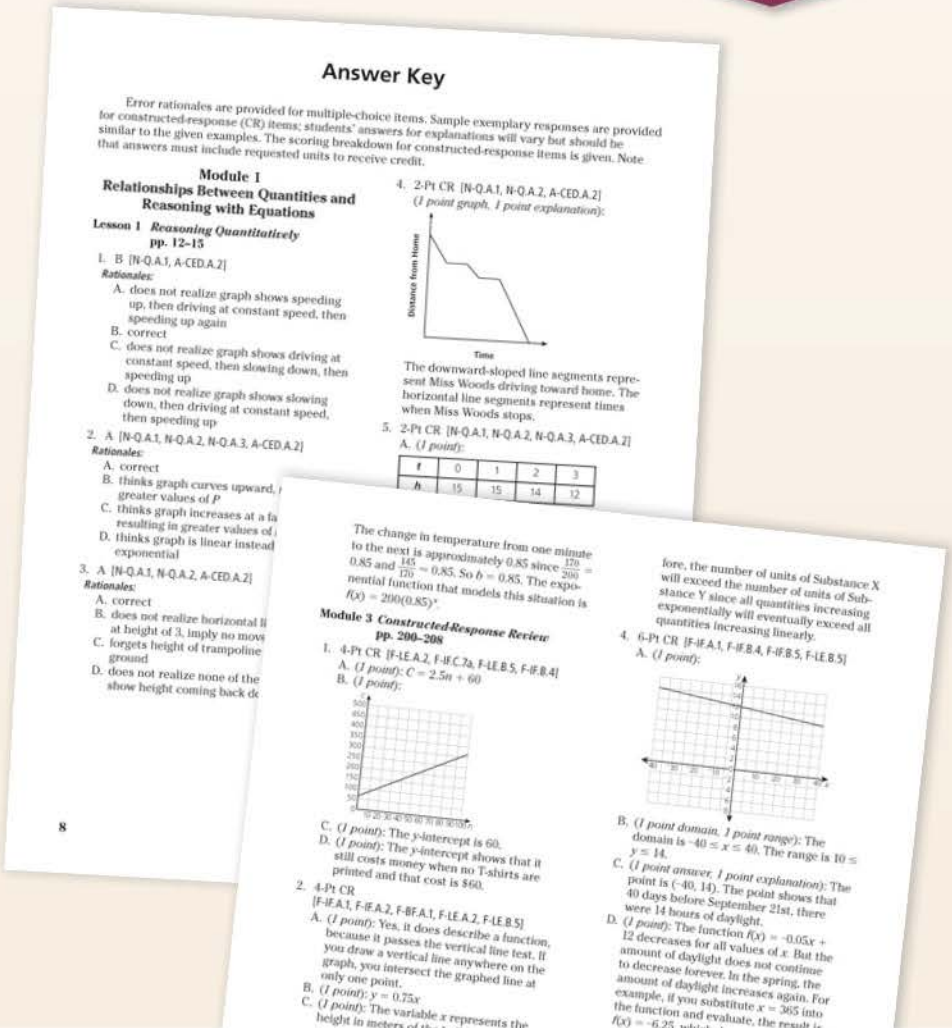
Quadratic Formula
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ where $a \neq 0$

Arithmetic Sequence
$a_n = a_1 + d(n - 1)$

Geometric Sequence
$a_n = a_1 r^{n-1}$

Teacher's Guide

- Suggestions for use
- NYS Common Core Learning Standards for Algebra I
- Reproducible formula reference sheet
- Answer key with
  - Standards alignment
  - Sample exemplary responses for constructed-response items
  - Error rationales for multiple-choice items
  - Scoring breakdown for constructed-response items



Answer Key

Error rationales are provided for multiple-choice items. Sample exemplary responses are provided for constructed-response (CR) items; students' answers for explanations will vary but should be similar to the given examples. The scoring breakdown for constructed-response items is given. Note that answers must include requested units to receive credit.

Module 1
Relationships Between Quantities and Reasoning with Equations

Lesson 1 ***Reasoning Quantitatively***
pp. 12–15

1. B [N-Q.A.1, A-CED.A.2]
Rationales:
A. does not realize graph shows speeding up, then driving at constant speed, then speeding up again
B. correct
C. does not realize graph shows driving at constant speed, then slowing down, then speeding up
D. does not realize graph shows slowing down, then driving at constant speed, then speeding up

2. A [N-Q.A.1, N-Q.A.2, N-Q.A.3, A-CED.A.2]
Rationales:
A. correct
B. thinks graph curves upward, greater values of *P*
C. thinks graph increases at a fa resulting in greater values of
D. thinks graph is linear instead exponential

3. A [N-Q.A.1, N-Q.A.2, A-CED.A.2]
Rationales:
A. correct
B. does not realize horizontal li at height of 3, imply no move
C. forgets height of trampoline ground
D. does not realize none of the show height coming back d

4. 2-Pt CR [N-Q.A.1, N-Q.A.2, A-CED.A.2]
(1 point graph, 1 point explanation):

The downward-sloped line segments represent Miss Woods driving toward home. The horizontal line segments represent times when Miss Woods stops.

5. 2-Pt CR [N-Q.A.1, N-Q.A.2, N-Q.A.3, A-CED.A.2]
A. *(1 point):*

t	0	1	2	3
h	15	15	14	12

The change in temperature from one minute to the next is approximately 0.85 since $\frac{170}{200} = 0.85$ and $\frac{145}{170} \approx 0.85$. So $b = 0.85$. The exponential function that models this situation is $f(x) = 200(0.85)^x$.

Module 3 ***Constructed-Response Review***
pp. 200–208

1. 4-Pt CR [F-LE.A.2, F-IF.C.7a, F-LE.B.5, F-IF.B.4]
A. *(1 point):* $C = 2.5n + 60$
B. *(1 point):*

C. *(1 point):* The *y*-intercept is 60.
D. *(1 point):* The *y*-intercept shows that it still costs money when no T-shirts are printed and that cost is $60.

2. 4-Pt CR [F-IF.A.1, F-IF.A.2, F-BF.A.1, F-LE.A.2, F-LE.B.5]
A. *(1 point):* Yes, it does describe a function, because it passes the vertical line test. If you draw a vertical line anywhere on the graph, you intersect the graphed line at only one point.
B. *(1 point):* $y = 0.75x$
C. *(1 point):* The variable *x* represents the

fore, the number of units of Substance X will exceed the number of units of Substance Y since all quantities increasing exponentially will eventually exceed all quantities increasing linearly.

4. 6-Pt CR [F-IF.A.1, F-IF.B.4, F-IF.B.5, F-LE.B.5]
A. *(1 point):*

B. *(1 point domain, 1 point range):* The domain is $-40 \le x \le 40$. The range is $10 \le y \le 14$.
C. *(1 point answer, 1 point explanation):* The point is (−40, 14). The point shows that 40 days before September 21st, there were 14 hours of daylight.
D. *(1 point):* The function $f(x) = -0.05x +$ 12 decreases for all values of *x*. But the amount of daylight does not continue to decrease forever. In the spring, the amount of daylight increases again. For example, if you substitute $x = 365$ into the function and evaluate, the result is

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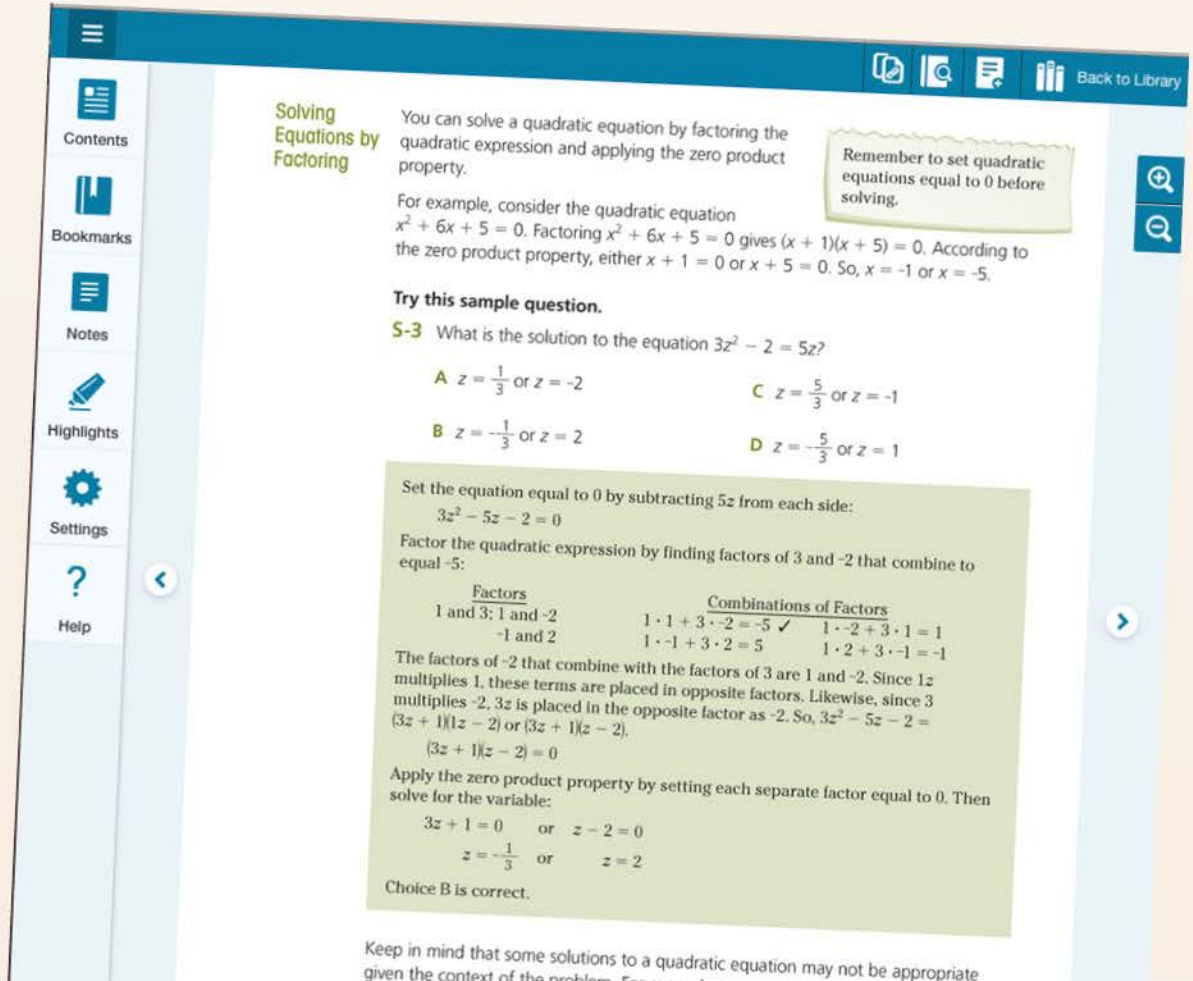

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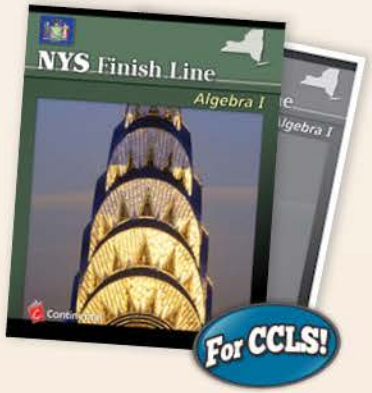

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