



OBJECTIVE

To understand the use of the commutative, associative, distributive, and identity properties and how they connect to division and multiplication

FOCUS LESSON

The Focus Lesson question leads students to use the distributive property to multiply a two-digit number by a one-digit number. They must recognize that a number can always be broken into the sum of two other numbers, and that they can use this to break a multiplication problem into smaller pieces to make it easier to solve. By breaking 14 into the sum of two one-digit numbers, such as 8 and 6, they can use their knowledge of basic multiplication facts to find the product of 14 and 4.

GUIDED PRACTICE

Students must recognize that the commutative property of multiplication allows for two factors to be multiplied in any order. Therefore, an array that shows 2 rows of 4 counters is the same as the one that shows 4 rows of 2 counters. Both show a total of 8 counters.



Lesson 4 **Properties of Operations**

Focus Lesson

The **commutative property of multiplication** states that the order of factors does not change the product. The commutative property is **not** true for division.
 $6 \times 3 = 18$ $3 \times 6 = 18$

The **associative property of multiplication** states that when the grouping of factors is changed, the product stays the same. The associative property is **not** true for division.
 $7 \times (3 \times 4)$ is the same as $(7 \times 3) \times 4$
 $7 \times 12 = 84$ $21 \times 4 = 84$

The **distributive property** states that you can multiply a sum by multiplying each addend separately and then adding the products.
 $5 \times 7 = 35$ $5 \times (2 + 5) = (5 \times 2) + (5 \times 5) = 10 + 25 = 35$

The **identity property** states that any number multiplied by 1 is that number. Any number divided by 1 is that number.
 $5 \times 1 = 5$ $5 \div 1 = 5$

Use the distributive property to help you multiply. **3.OA.5**

$14 \times 4 = \underline{56}$

- Break 14 into two one-digit numbers. **8 and 5** *Answers may vary.*
- Rewrite the problem using the distributive property. **$(9 \times 4) + (5 \times 4)$**
- What is the value of the first set of parentheses? **36**
- What is the value of the second set of parentheses? **20**
- What operation do you use to combine the two values from steps 3 and 4?
addition
- Write a number sentence to show step 5. Find the answer.
 $36 + 20 = 56$

What is the product of 14×4 ?

© The Continental Press, Inc. DUPLICATING THIS MATERIAL IS ILLEGAL. **Operations and Algebraic Thinking UNIT 1 23**

Vocabulary

Associative Property: allows grouping of numbers with parentheses to be added or multiplied: $a + (b + c) = (a + b) + c$ and $a \times (b \times c) = (a \times b) \times c$

Commutative Property: allows numbers to be added or multiplied in any order: $a + b = b + a$ and $a \times b = b \times a$

Distributive Property: allows a number to be multiplied by a sum or each addend to be multiplied separately and the products added: $a(b + c) = ab + ac$

Identity Property: states that any number multiplied by 1 is always that number; any number divided by 1 is always that number

Common Core Learning Standards

3.OA.5: Apply properties of operations as strategies to multiply and divide.

3.OA.6: Understand division as an unknown-factor problem.

Guided Practice

Jammy wrote the number sentence $4 \times 2 = 8$. She drew a matching array shown below. **3.OA.5**

Use the commutative property to write a number sentence with the same product. Draw an array to match the new number sentence.

$2 \times 4 = 8$

- What does the commutative property say?
You can switch the order of factors and the product does not change.
- How can you change the array to show the new multiplication sentence?
Make it show 2 rows of 4 counters.

Write the new number sentence. Draw the new array.

© The Continental Press, Inc. DUPLICATING THIS MATERIAL IS ILLEGAL. **24 UNIT 1 Operations and Algebraic Thinking**

Independent Practice

Solve the following problems.

- 1 Samuel and Orlando are working on the problem shown. **3.OA.5**
 $5 \times (3 \times 4) = \square$ **DOK: 3**

Orlando rewrites the problem as $(5 \times 4) \times 3 = \square$. Samuel rewrites the problem as $5 \times (4 \times 3) = \square$. Will either student still get the problem correct? Explain. Show your work for all three problems and solve.

**Yes, they will both get the correct answer. The associative and commutative properties let you change the order of numbers being multiplied: $5 \times (3 \times 4) = 5 \times 12 = 60$
 $(5 \times 4) \times 3 = 20 \times 3 = 60$ $5 \times (4 \times 3) = 5 \times 12 = 60$**

- 2 Place a check mark in the box next to the number sentences that are true. Mark all that apply. **3.OA.5, 6**
DOK: 2
- $5 \times 1 = 5$
 - $4 + 1 = 1$
 - $6 \times 4 = 4 \times 6$
 - $2 \times 5 = 2 + 5$
 - $27 \times 5 = (20 \times 5) + (7 \times 5)$
 - $22 \times 3 = (12 \times 2) + (10 \times 1)$

ANSWER RATIONALE

- Multiplication of all three number sentences should result in 60. Orlando writes the problem using both the commutative property (to change the order of 4 and 3) and the associative property (to move the parentheses). Samuel uses the commutative property to change the order of 3 and 4. The product for all three number sentences is 60.
- The first number sentence illustrates the identity property. The third number sentence illustrates the commutative property. The fifth number sentence illustrates the distributive property. Selecting the second number sentence shows a misunderstanding of the identity property and how it relates to division. Selecting the fourth number sentence shows a misunderstanding of the commutative property. Selecting the sixth number sentence shows a misunderstanding of the distributive property and how to break apart a number.

Independent Practice

- 3 Billy earns \$22 each day for 5 days. Which equation will show how much Billy earns for the 5 days? Mark all that apply. **3.OA.5**
DOK: 2

- A $(22 \times 5) + (22 \div 5)$
- B $(20 \times 2) + (2 + 3)$
- C $(20 \times 5) + (2 \times 5)$
- D $(10 \times 5) + (12 \times 5)$
- E $(20 \times 5) \times (2 \times 5)$
- F $(17 + 5) \times (5 + 5)$

- 4 Look at the multiplication problem below. **3.OA.5**
 $6 \times (3 \times 4)$ **DOK: 2**
- Choose the ways to rewrite the problem and still get the correct answer. Mark all that apply.

- $6 \times (4 \times 3)$
- $4 \times (3 \times 6)$
- $6 + (3 \times 4)$
- $4 \times (3 + 6)$
- $4 + 6 + 3$
- $4 \times 3 \times 6$

- Options C and D are correct answers. Both correctly utilize the distributive property to break 22 into smaller numbers and multiply the smaller numbers by 5. Option A uses the same numbers as the word problem, but does not use the distributive property correctly. Option B shows a misunderstanding of breaking the numbers apart. Options E and F show a misunderstanding of the operations in the distributive property.
- The equations $6 \times (4 \times 3)$, $4 \times (3 \times 6)$, and $4 \times 3 \times 6$ will still get the correct answer. Students who did not choose all three options do not fully understand that the order or grouping of numbers does not matter in multiplication. Students who chose $6 + (3 \times 4)$, $4 \times (3 + 6)$, or $4 + 6 + 3$ do not understand that addition will change the outcome of the answer even though the numbers are the same. Students who chose the first two options but not the last option do not understand that you will still receive the same answer without the parentheses.

- 5 Options A and C show the identity property with division and multiplication. Option D shows the commutative property, and option F shows the associative property. Option B incorrectly shows the identity property by switching the quotient and the divisor. Option E does not show a correct number sentence.
- 6 Rob uses the commutative property correctly, changing the order of the factors in a multiplication sentence. Cindy misuses the commutative property in a division sentence. The commutative property is not true for division or subtraction.
- 7 Use the distributive property to combine numbers. Cara has broken a larger number apart. So her original expression was the product of $(20 + 7)$ and 7 or 27×7 . An incorrect answer of 189 shows that the student misread the question and found the value of the expression.
- 8 The identity property says that a number multiplied by 1 is that number. Since each bag of coffee is 1 pound, the number of bags (9) is multiplied by 1 for a total of 9 pounds.

Hands-On Extension Activity

Make a spinner divided into three equal parts and label the parts *Distributive*, *Commutative*, and *Associative*. Give each pair or group of students a multiplication expression involving a one-digit number \times two-digit number, e.g., 6×48 . Direct them to spin the arrow on the spinner and then rewrite the expression using the property of operations the arrow stops on. For example, for distributive, $6 \times 48 = 6 \times (40 + 8)$. Spin the arrow again, and rewrite the last expression according to the new property. For example, for commutative, $6 \times (40 + 8)$ could be written $(40 + 8) \times 6$. Be sure students understand that whatever is inside parentheses can be thought of as a single number. Continue playing the game and rewriting the expression in as many ways as possible.

Independent Practice

5 Match the equation with its property by placing the letter in the correct column of the table. Not all choices will be used.

**3.OA.5, 6
DOK: 2**

A $5 \div 1 = 5$
 B $2 \times 2 = 1$
 C $1 \times 5 = 5$
 D $4 \times 2 = 2 \times 4$
 E $6 \times 4 = 4 + 6$
 F $(3 \times 1) \times 4 = 3 \times (1 \times 4)$

Identity Property	Commutative Property	Associative Property
A C	D	F

© The Continental Press, Inc. Duplicating this material is illegal. Operations and Algebraic Thinking UNIT 1 27

Independent Practice

6 Rob writes the following two number sentences.
 $2 \times 4 = 8$ and $4 \times 2 = 8$
 Cindy writes the following two number sentences.
 $4 \div 2 = 2$ and $2 \div 4 = 2$
 Are both students correct? Explain.
 Rob is correct [incorrect] because **Rob shows the commutative property. He switches the order of the factors. The product will still be the same.**
 Cindy is [correct, incorrect] because **Cindy thinks the commutative property works on division, too. But you can't switch the order of the numbers in division.**

7 Cara is multiplying two numbers. She uses the distributive property to break a large number into two smaller numbers. The equation she has now is $(20 \times 7) + (7 \times 7)$. What was Cara's original equation?
 27×7
**3.OA.5
DOK: 2**

8 A bag of coffee weighs 1 pound. Write a multiplication sentence that will show how many pounds 9 bags weigh.
 $9 \times 1 = 9$
 or $1 \times 9 = 9$
**3.OA.5
DOK: 2**

28 UNIT 1 Operations and Algebraic Thinking © The Continental Press, Inc. Duplicating this material is illegal.