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Objective

To distinguish between linear and nonlinear functions and to solve problems using linear functions

1 Introduction

Review linear equations, focusing on equations in slope-intercept form. Define a linear function as any function for which the graph is a straight line. A linear function has an equation that can be written in slope-intercept form. Identify the linear function from the given examples. Work through the second sample item to help students use a table to determine the equation of a function and then graph the function. Review how to use the slope formula.

Think About It

Students should recognize real-life situations that can be represented with a linear relationship. For example, the relationship between the number of pounds of apples and the total cost is a linear relationship.

Common Core State Standards

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Vocabulary

linear function: a function that has the same change in y-values for each change in x-values. Its graph is a straight line.

slope: the steepness of a line that shows how the change in one variable relates to the change in the other variable

***y*-intercept:** the point $(0, b)$ where a line intersects the y-axis

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LESSON 18 **Linear Functions** CCSS: 8.F.3, 4

1 Introduction

A **linear function** is any function with a graph that is a straight line. The equation for a linear function can be written in slope-intercept form, $y = mx + b$, where m is the **slope** and b is the ***y*-intercept.** The slope is the rate of change or how steep the line is. Slope is shown as the ratio of the change in y to the change in x. The y-intercept is where the graph crosses the y-axis at the point $(0, b)$.

Which of these equations is a linear function?

Equation 1: $y = 2x^2 + 3$ Equation 3: $y = x^3$

Equation 2: $-5x + y = 1$ Equation 4: $y = \frac{3}{x - 1}$

In equations 1 and 3, x is raised to a power, so these equations are not linear. In equation 4, x appears in the denominator, so it is not linear. Equation 2 can be written in slope-intercept form by adding $5x$ to both sides.

$$-5x + y = 1$$
$$-5x + 5x + y = 5x + 1$$
$$y = 5x + 1$$

Equation 2 is a linear equation.

You can write any linear equation in slope-intercept form by isolating y using inverse operations.

A function is nonlinear if one of the variables is raised to a power, inside absolute value symbols, or in the denominator of a fraction when solved for one of the variables.

You can use an input-output table, a graph, or two points on a line to write a linear equation.

Write a linear equation from the values in the table.

x	-3	0	3	6	9
y	-1	5	11	17	23

To determine the slope, pick any two points and use the slope formula. Use (0, 5) and (-3, -1).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{0 - (-3)} = \frac{5 + 1}{0 + 3} = \frac{6}{3} = 2$$

When using the slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, make sure to pick points that are ordered pairs.

To determine the y-intercept, find the value of y when x is 0. In the table, the value of y is 5 when x is 0. The slope is 2 and the y-intercept is 5. So, $m = 2$ and $b = 5$.

The equation is $y = 2x + 5$.

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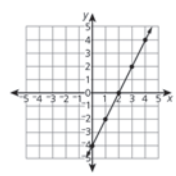
Write a linear equation from the graph.



The graph crosses the y-axis at -4, so the y-intercept is -4.

To find the rate of change, pick any two points and use the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{4 - 3} = \frac{2}{1} = 2$$

So, the equation is $y = 2x - 4$.

Think About It

What is an example of a linear relationship in your daily life?

2 Focused Instruction

Many real-life situations can be modeled with a linear function. Think about the relationship between the variables and the constants in the situation.

➤ The Parker family went camping. The campground charged an entrance fee of \$20 and \$10 per night. Write a function to determine the total cost, y, for a camping trip for x nights.

How much does the campground charge per night? $10

How much does the campground charge for entrance? $20

What does x represent in this problem? the number of nights spent camping

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Focused Instruction Lesson 18

Write an expression to represent the cost for x nights of camping only. 10x

Is the entrance fee a constant or does it change? constant

A constant is a value that does not change.

Is the entrance fee added to or subtracted from the cost per night to camp? added to

What does y represent in this problem? the total cost for a camping trip

Think about the slope-intercept form of an equation. Which value represents m in this problem? 10 Which value represents b? 20

The slope-intercept form for a linear equation is $y = mx + b$.

Write an equation in slope-intercept form representing the total cost for x nights of camping. $y = 10x + 20$

Use information in a situation to find the slope and write an equation for a linear function.

➤ A candle is 12 inches tall. After it burns for 4 hours, it is 9 inches tall. Cam graphs the relationship between the height of the candle, y, and the amount of time it burns, x.

What is the initial height of the candle? 12 inches

What is the value of y when x is 0? 12 Write this as an ordered pair. (0, 12)

What is the value of y when x is 4? 9 Write this as an ordered pair. (4, 9)

Sometimes you must find ordered pairs for points in the information in a problem.

Use the two points you found above to find the slope of this linear function. $\frac{12-9}{0-4} = -\frac{3}{4}$

Use the slope and the y-intercept to write the equation describing this linear function. $y = -\frac{3}{4}x + 12$

Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

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Focused Instruction Lesson 18

Use information in a table to determine if a function is linear.

➤ Zachary completed a science project to determine the height (h) of a helium balloon over time (t). The results of the experiment are shown in the table.

Time, t (in seconds)	0	5	10	20	25
Height, h (in feet)	0	12	16	19	18

Which value represents x? t

Which value represents y? h

Does time change in a constant pattern?
Yes, time increases by 5 seconds in each entry.

Does height change in a constant pattern?
No, it changes by different amounts each time.

Is the rate of change constant? no

Is this a linear relationship? no

Use what you know about functions to solve these problems.

1 Circle the equation that represents a linear function.

$y = \sqrt{x+2}$ $3x + 4y^2 = 9$ $y = \frac{x}{6} + 2$ $y = \frac{1}{2}x^2$

2 Find the slope and y-intercept for the function $y = \frac{2}{3}x - \frac{1}{2}$.

slope: $\frac{2}{3}$; y-intercept: $-\frac{1}{2}$

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PART 2 Focused Instruction

Students will write linear equations to describe real-life situations. In the first problem, students must decide which values in the situation are variables and which are constants. In the second problem, students use the information given to find two points that would appear on the graph of the function. They use the points to find the slope and then write the equation for the function. In the third problem, students use information in a table to determine if a function is linear or nonlinear.

Conclude the Focused Instruction section by having students answer two questions about linear functions.

Connections to Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

Part 3 Guided Practice

Students should complete the Guided Practice section on their own. Offer assistance as needed, pointing out the reminder and hint boxes along the right side of the page.

Part 4 Independent Practice Answer Rationales

1 **PART A** The formula comparing temperature is written in the slope-intercept form, $y = mx + b$. So 1.8 is m, the slope or rate of change.

PART B The Fahrenheit temperature when the Celsius temperature is 0° is the same as the y-intercept. When solving for the y-intercept, set x, or C, equal to 0: $F = (1.8)(0) + 32$; $F = 0 + 32 = 32$.

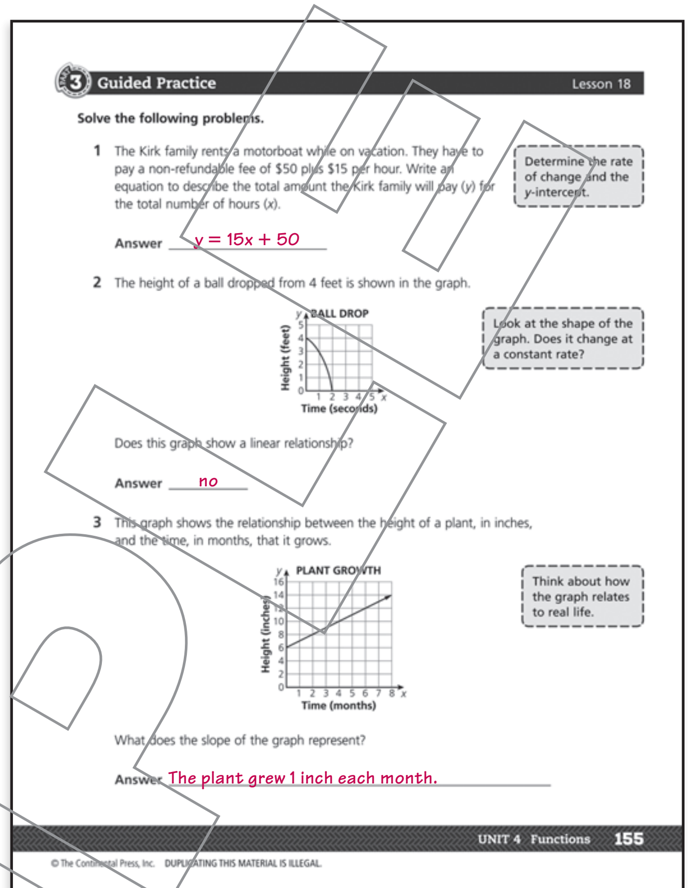
PART C Substitute the value of C to find F:

°C	°F
−20	$F = (1.8)(-20) + 32$ $F = -36 + 32$ $F = -4$
−10	$F = (1.8)(-10) + 32$ $F = -18 + 32$ $F = 14$
0	$F = (1.8)(0) + 32$ $F = 0 + 32$ $F = 32$
25	$F = (1.8)(25) + 32$ $F = 45 + 32$ $F = 77$
50	$F = (1.8)(50) + 32$ $F = 90 + 32$ $F = 122$

PART D A linear relationship is in the form $y = mx + b$, where m is the slope and b is the y-intercept. The formula comparing temperatures is in this form, which means that it is a linear relationship.

2 A table that shows a linear relationship has a constant change. Choice A is represented by the equation $y = x^2$, which is not a linear relationship. Choice B has a constant change of 3 because each y-value is 3 more than the corresponding x-value.

In choice C, the slope between two points is not consistent: $\frac{15 - 10}{5 - 2} = \frac{5}{3}$; $\frac{19 - 15}{8 - 5} = \frac{4}{3}$. In choice D, the relationship is modeled by the equation $y = \sqrt{x}$, so it is not a linear function. Choice B is correct.



Part 3 Guided Practice — Lesson 18

Solve the following problems.

1 The Kirk family rents a motorboat while on vacation. They have to pay a non-refundable fee of $50 plus $15 per hour. Write an equation to describe the total amount the Kirk family will pay (y) for the total number of hours (x).

Determine the rate of change and the y-intercept.

Answer $y = 15x + 50$

2 The height of a ball dropped from 4 feet is shown in the graph.

Look at the shape of the graph. Does it change at a constant rate?

Does this graph show a linear relationship?

Answer no

3 This graph shows the relationship between the height of a plant, in inches, and the time, in months, that it grows.

Think about how the graph relates to real life.

What does the slope of the graph represent?

Answer The plant grew 1 inch each month.

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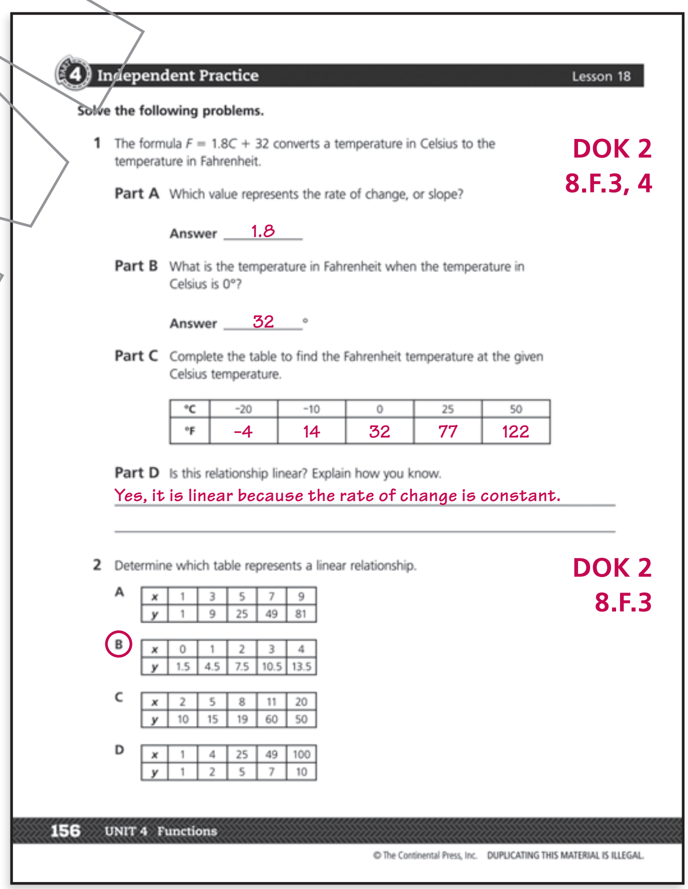

Part 4 Independent Practice — Lesson 18

Solve the following problems.

1 The formula $F = 1.8C + 32$ converts a temperature in Celsius to the temperature in Fahrenheit.

DOK 2
8.F.3, 4

Part A Which value represents the rate of change, or slope?

Answer 1.8

Part B What is the temperature in Fahrenheit when the temperature in Celsius is 0°?

Answer 32 °

Part C Complete the table to find the Fahrenheit temperature at the given Celsius temperature.

°C	−20	−10	0	25	50
°F	−4	14	32	77	122

Part D Is this relationship linear? Explain how you know.

Yes, it is linear because the rate of change is constant.

2 Determine which table represents a linear relationship.

DOK 2
8.F.3

A

x	1	3	5	7	9
y	1	9	25	49	81

B

x	0	1	2	3	4
y	1.5	4.5	7.5	10.5	13.5

C

x	2	5	8	11	20
y	10	15	19	60	50

D

x	1	4	25	49	100
y	1	2	5	7	10

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4 Independent Practice Lesson 18

3 This function table shows the relationship between the number of hours, *h*, an auto mechanic works to repair a car and the total cost, *C*, in dollars, the mechanic charges.

DOK 3
8.F.4

h	*C*($)
1	110
2	170
3	230
4	290

Part A Write an equation that models this relationship. Explain how you found your answer.

C = 60*h* + 50; I found the rate of change and then substituted 0 for *h* to find the *y*-intercept.

Part B What is the value of *C*, in dollars, when $h = 2\frac{1}{4}$? Explain how you know.

I found the value of *C* by substituting $2\frac{1}{4}$ for *h*:
C = 60(2.25) + 50 = (135) + 50 = $185.

4 Match the equations from the box with the correct description. You will not use all the equations.

DOK 2
8.F.4

$y - 15x = 0$	$15y = x$	$y = 150 + 50x$	$y = 150x + 50$

Description	Equation
Leonard makes $15 an hour.	$y - 15x = 0$
Alexander has $150 and plans to save $50 per month.	$y = 150 + 50x$

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4 Independent Practice Lesson 18

5 Which table of values matches the equation $y = -x + 3$?

DOK 2
8.F.4

A

x	*y*
2	-1
4	1
6	3
8	5

B

x	*y*
2	1
4	-1
6	-3
8	-5

C

x	*y*
2	5
4	7
6	9
8	11

D

x	*y*
2	-5
4	-7
6	-9
8	-11

6 What is the slope of a line that passes through the points (2, 5) and (9, 26)?

DOK 2
8.F.3, 4

Answer 3

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3 PART A To write an equation in the form $y = mx + b$, find the rate of change for *x* by using two points and the slope formula. In this relationship, *x* is represented by *h* and *y* is represented by *C*. Using the points (1, 110) and (2, 170), find the slope: $\frac{170 - 110}{2 - 1} = \frac{60}{1} = 60$. The slope is 60. Find *b* by substituting two given values and solving for *b*: $110 = 60(1) + b$; $110 = 60 + b$; $50 = b$. The *y*-intercept is 50, so the equation is $C = 60h + 50$.

PART B Use the equation from Part A and solve for *C* when *h* is equal to $2\frac{1}{4}$: $C = 60\left(2\frac{1}{4}\right) + 50$; $C = 135 + 50$; $C = 185$. The cost is $185 when the auto mechanic works for $2\frac{1}{4}$ hours.

4 To write an equation, determine which number is the rate of change and which number is the initial value. A function can be written in the form of $y = mx + b$. If Leonard makes $15 per hour, the relationship is modeled by the equation $y = 15x$. Look for an equation equivalent to this: $y - 15x = 0$. Alexander has $150, so this is the *y*-intercept. The savings increases by $50 per month, or $50*x*. This relationship is shown by the equation $y = 50x + 150$, or $y = 150 + 50x$.

5 To find the table that matches the equation, check whether the outputs in each table are able to correctly substitute in the linear equation to equal the input. In choice A, $-1 \neq -2 + 3$, so it is not correct. In choice C, $5 \neq -2 + 3$, so it is not correct. In choice D, $-5 \neq -2 + 3$, so it is not correct. The only table that matches the equation is choice B: $1 = -2 + 3$; $-1 = -4 + 3$; $-3 = -6 + 3$; $-5 = -8 + 3$.

6 To find the rate of change, use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ with the given points: $\frac{26 - 5}{9 - 2} = \frac{21}{7} = 3$. The slope of the line is 3.

Extension Activity

Distribute blank coordinate planes to students. Have each student draw a line on the coordinate plane (lines must intersect the *y*-axis). Students should exchange graphs with a partner and find the slope and *y*-intercept of the graph. They should then write the equation of the line in slope-intercept form.