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10 Solving Problems with Proportional Relationships

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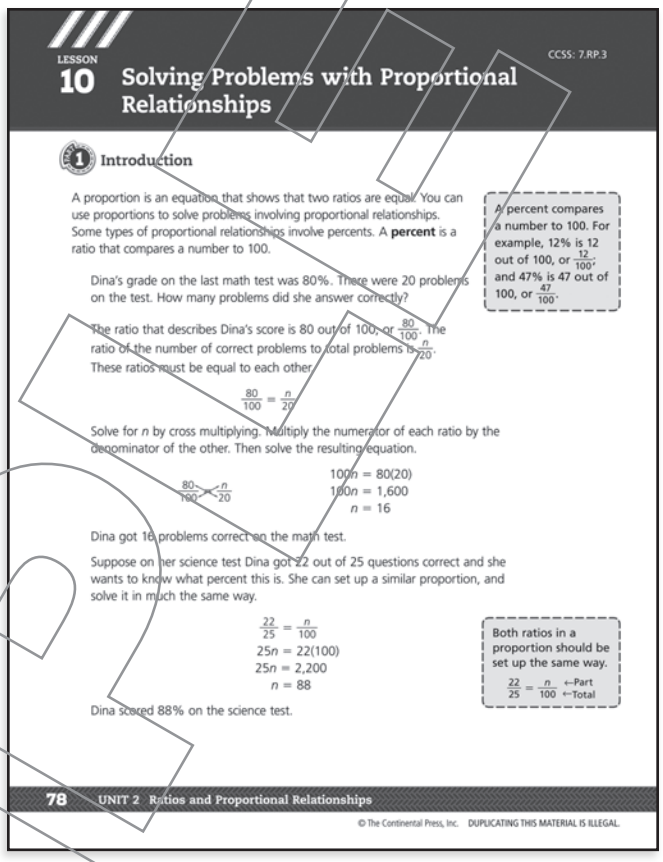
Objective

To use proportions to solve real-world percent problems, including discounts and tax

PART 1 Introduction

Review that a percent is a ratio that compares a number to 100. Discuss problem situations where percents are used: finding a percent of a number, finding what percent one number is of another, and finding the whole when a percent is known. Then work through the examples that show how to set up a representative proportion and solve for the unknown to find a percent of a number and to find the percent one number is of another. Be sure students understand that one ratio of the proportion will always be some number to 100, which represents the percent.



LESSON 10 Solving Problems with Proportional Relationships

CCSS: 7.RP.3

1 Introduction

A proportion is an equation that shows that two ratios are equal. You can use proportions to solve problems involving proportional relationships. Some types of proportional relationships involve percents. A **percent** is a ratio that compares a number to 100.

A percent compares a number to 100. For example, 12% is 12 out of 100, or $\frac{12}{100}$, and 47% is 47 out of 100, or $\frac{47}{100}$.

Dina's grade on the last math test was 80%. There were 20 problems on the test. How many problems did she answer correctly?

The ratio that describes Dina's score is 80 out of 100, or $\frac{80}{100}$. The ratio of the number of correct problems to total problems is $\frac{n}{20}$. These ratios must be equal to each other.

$$\frac{80}{100} = \frac{n}{20}$$

Solve for n by cross multiplying. Multiply the numerator of each ratio by the denominator of the other. Then solve the resulting equation.

$\frac{80}{100} \times \frac{n}{20}$

$100n = 80(20)$
$100n = 1{,}600$
$n = 16$

Dina got 16 problems correct on the math test.

Suppose on her science test Dina got 22 out of 25 questions correct and she wants to know what percent this is. She can set up a similar proportion, and solve it in much the same way.

$$\frac{22}{25} = \frac{n}{100}$$
$$25n = 22(100)$$
$$25n = 2{,}200$$
$$n = 88$$

Dina scored 88% on the science test.

Both ratios in a proportion should be set up the same way.
$\frac{22}{25} = \frac{n}{100}$ ←Part ←Total

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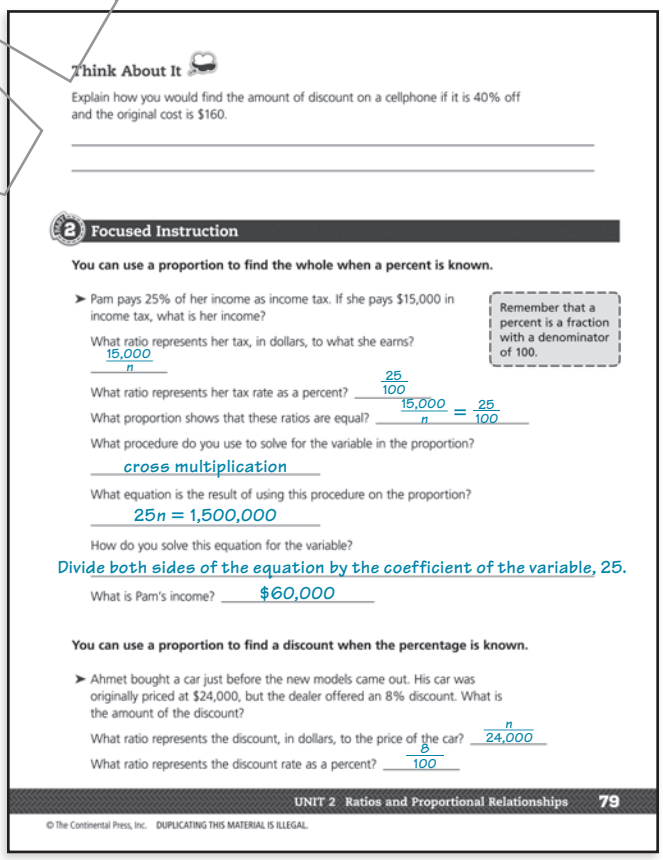
Think About It

Students should recognize that if a discount is 40% of the original price, then the sale price is the remaining amount, which is 60% of the original price. Either 40% can be subtracted from $160 or 60% of $160 can be found.



Think About It

Explain how you would find the amount of discount on a cellphone if it is 40% off and the original cost is $160.

2 Focused Instruction

You can use a proportion to find the whole when a percent is known.

➤ Pam pays 25% of her income as income tax. If she pays $15,000 in income tax, what is her income?

Remember that a percent is a fraction with a denominator of 100.

What ratio represents her tax, in dollars, to what she earns? $\frac{15{,}000}{n}$

What ratio represents her tax rate as a percent? $\frac{25}{100}$

What proportion shows that these ratios are equal? $\frac{15{,}000}{n} = \frac{25}{100}$

What procedure do you use to solve for the variable in the proportion?

cross multiplication

What equation is the result of using this procedure on the proportion?

$25n = 1{,}500{,}000$

How do you solve this equation for the variable?

Divide both sides of the equation by the coefficient of the variable, 25.

What is Pam's income? $60,000

You can use a proportion to find a discount when the percentage is known.

➤ Ahmet bought a car just before the new models came out. His car was originally priced at $24,000, but the dealer offered an 8% discount. What is the amount of the discount?

What ratio represents the discount, in dollars, to the price of the car? $\frac{n}{24{,}000}$

What ratio represents the discount rate as a percent? $\frac{8}{100}$

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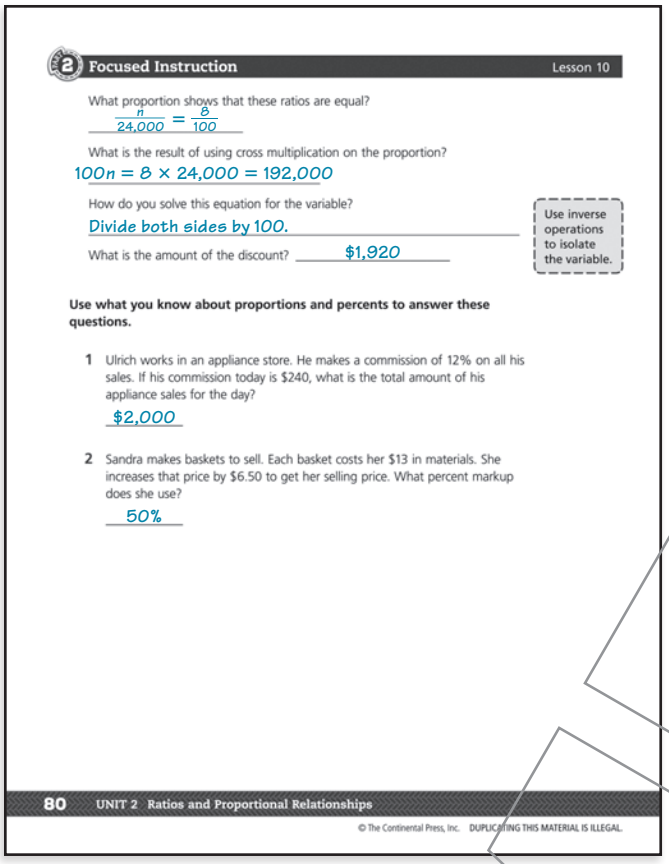
Common Core State Standard

7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.

Vocabulary

percent: a ratio that compares a number to 100, written with the symbol %



Focused Instruction Lesson 10

What proportion shows that these ratios are equal?

$\frac{n}{24{,}000} = \frac{8}{100}$

What is the result of using cross multiplication on the proportion?

$100n = 8 \times 24{,}000 = 192{,}000$

How do you solve this equation for the variable?

Divide both sides by 100.

What is the amount of the discount? $1,920

Use inverse operations to isolate the variable.

Use what you know about proportions and percents to answer these questions.

1 Ulrich works in an appliance store. He makes a commission of 12% on all his sales. If his commission today is $240, what is the total amount of his appliance sales for the day?

$2,000

2 Sandra makes baskets to sell. Each basket costs her $13 in materials. She increases that price by $6.50 to get her selling price. What percent markup does she use?

50%

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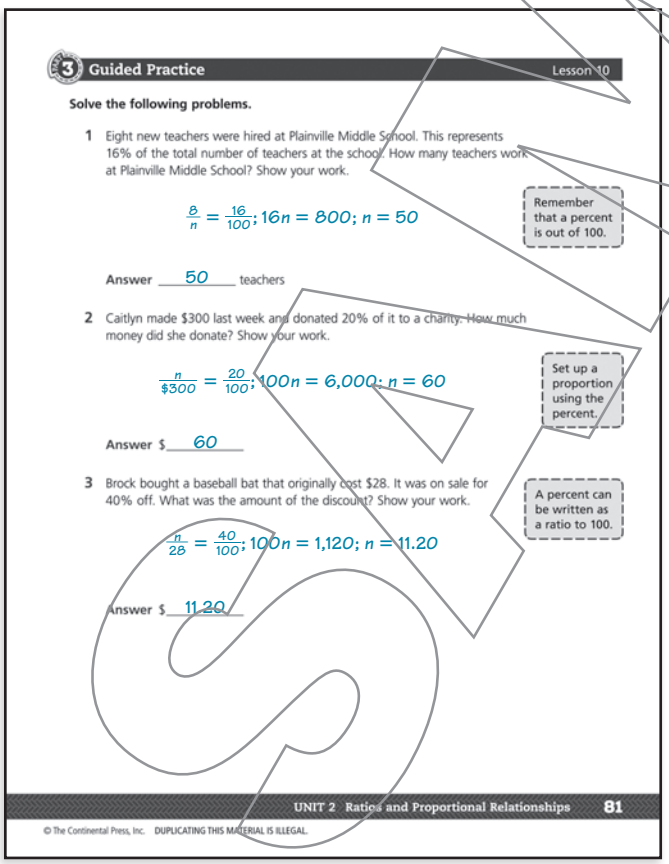

Guided Practice Lesson 10

Solve the following problems.

1 Eight new teachers were hired at Plainville Middle School. This represents 16% of the total number of teachers at the school. How many teachers work at Plainville Middle School? Show your work.

$\frac{8}{n} = \frac{16}{100}; 16n = 800; n = 50$

Remember that a percent is out of 100.

Answer 50 teachers

2 Caitlyn made $300 last week and donated 20% of it to a charity. How much money did she donate? Show your work.

$\frac{n}{\$300} = \frac{20}{100}; 100n = 6{,}000; n = 60$

Set up a proportion using the percent.

Answer $ 60

3 Brock bought a baseball bat that originally cost $28. It was on sale for 40% off. What was the amount of the discount? Show your work.

$\frac{n}{28} = \frac{40}{100}; 100n = 1{,}120; n = 11.20$

A percent can be written as a ratio to 100.

Answer $ 11.20

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PART 2 Focused Instruction

First, students are given a percent and its value and asked to find the whole. Questions guide them in setting up the two ratios of the proportions; students recognize that $\frac{15{,}000}{n}$ represents the amount of tax paid on an income of n dollars, and this is equivalent to $\frac{25}{100}$. They cross multiply to achieve an equation that they can solve for the unknown.

Next, students identify a discount as a percent of a number. They use this understanding to set up ratios and a proportion that can be solved to find the amount of the discount.

Conclude the Focused Instruction section by having students solve two problems involving percents.

PART 3 Guided Practice

Students should complete the Guided Practice section on their own. Offer assistance as needed, pointing out the reminder and hint boxes along the right side of the page.

Connections to Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.
- Look for and make use of structure.

Independent Practice Answer Rationales

1 Because all the ratios are equal, any one of them can be used to set up the discount proportion. For example, $\frac{1.25}{5} = \frac{n}{100}$. Cross multiplying results in $5n = 100 \times 1.25 = 125$. Divide both sides of the equation by 5 to solve: $\frac{5n}{5} = \frac{125}{5}$, and $n = 25$ or 25%. Choice B is correct. Choice A is half of the discount percent, and choices C and D are too large.

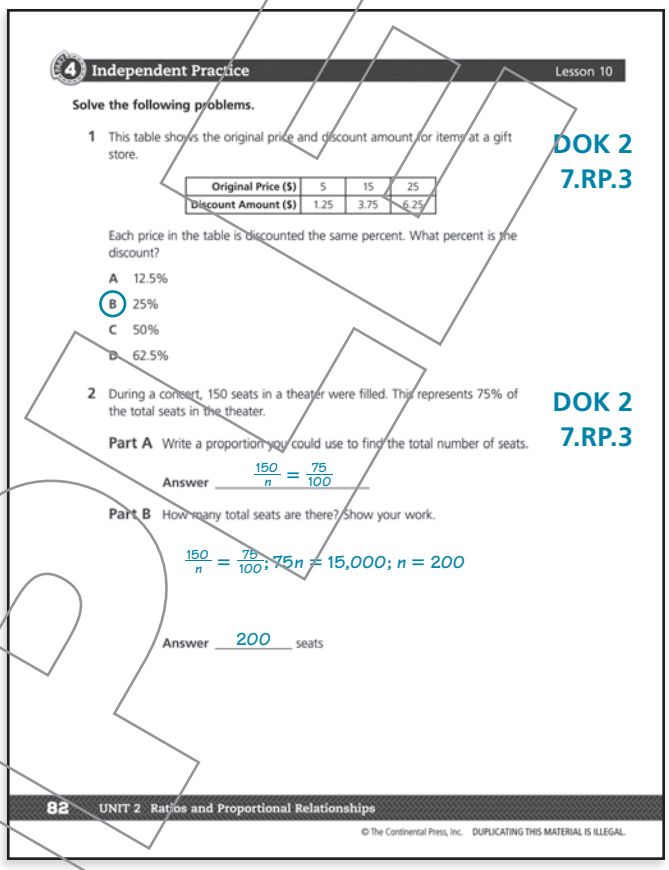
2 PART A Both ratios in the proportion compare the number of seats filled to the total number of seats: $\frac{150}{n} = \frac{75}{100}$.

PART B Cross multiply to solve for n: $75n = 150 \times 100 = 15{,}000$. Divide both sides of the equation by 75: $n = \frac{15{,}000}{75} = 200$. There are 200 seats in the theater.

3 The proportion uses ratios that compare the commission to the total, $\frac{n}{180{,}000} = \frac{4}{100}$. Cross multiply for $100n = 720{,}000$. Divide both sides by 100, and $n = 7{,}200$. The commission is $7,200; choice B is correct. Choice A divides 180,000 by 4 instead of multiplying the two numbers. Choice C divides instead of multiplies and has the decimal point in the wrong place. Choice D has the decimal in the wrong place.

4 The ratio used to solve the problem is $\frac{6}{n} = \frac{5}{100}$. Solving for n, $5n = 600$, so $\frac{5n}{5} = \frac{600}{5}$, and $n = 120$. The actual car is 120 inches long. Since the question asks for the length in feet, divide the length in inches by 12 for 10 feet.

5 PARTS A AND B A tax is a percent of a number, in this case, the cost of the television. The proportion that represents this situation is $\frac{16.25}{250} = \frac{n}{100}$. First cross multiply: $1{,}625 = 250n$. Then divide both sides by 250 for $n = 6.5$. The tax rate is 6.5%.



Independent Practice Lesson 10

Solve the following problems.

1 This table shows the original price and discount amount for items at a gift store. DOK 2 7.RP.3

Original Price ($)	5	15	25
Discount Amount ($)	1.25	3.75	6.25

Each price in the table is discounted the same percent. What percent is the discount?

A 12.5%
B 25%
C 50%
D 62.5%

2 During a concert, 150 seats in a theater were filled. This represents 75% of the total seats in the theater. DOK 2 7.RP.3

Part A Write a proportion you could use to find the total number of seats.

Answer $\frac{150}{n} = \frac{75}{100}$

Part B How many total seats are there? Show your work.

$\frac{150}{n} = \frac{75}{100}$; $75n = 15{,}000$; $n = 200$

Answer 200 seats

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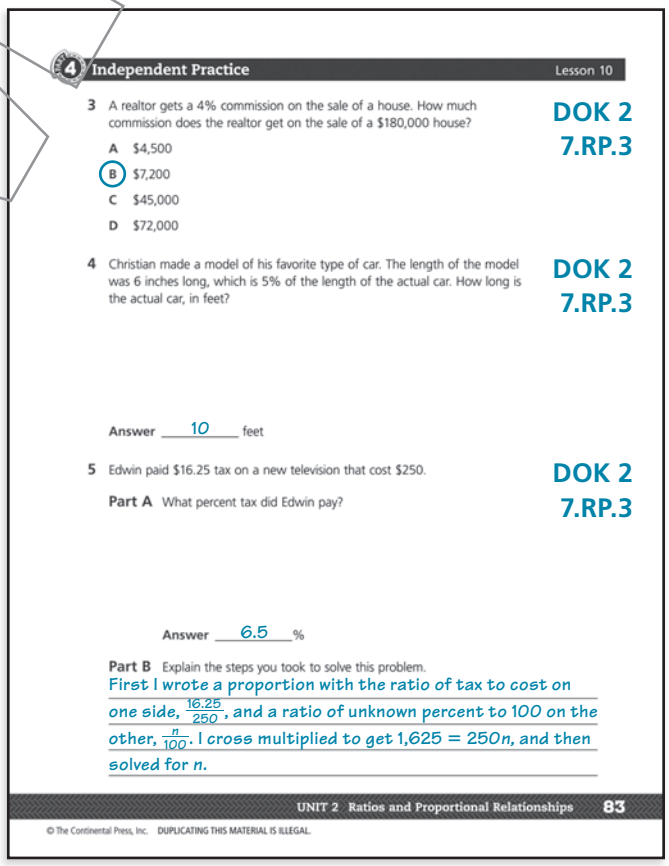

Independent Practice Lesson 10

3 A realtor gets a 4% commission on the sale of a house. How much commission does the realtor get on the sale of a $180,000 house? DOK 2 7.RP.3

A $4,500
B $7,200
C $45,000
D $72,000

4 Christian made a model of his favorite type of car. The length of the model was 6 inches long, which is 5% of the length of the actual car. How long is the actual car, in feet? DOK 2 7.RP.3

Answer 10 feet

5 Edwin paid $16.25 tax on a new television that cost $250. DOK 2 7.RP.3

Part A What percent tax did Edwin pay?

Answer 6.5 %

Part B Explain the steps you took to solve this problem.

First I wrote a proportion with the ratio of tax to cost on one side, $\frac{16.25}{250}$, and a ratio of unknown percent to 100 on the other, $\frac{n}{100}$. I cross multiplied to get $1{,}625 = 250n$, and then solved for n.

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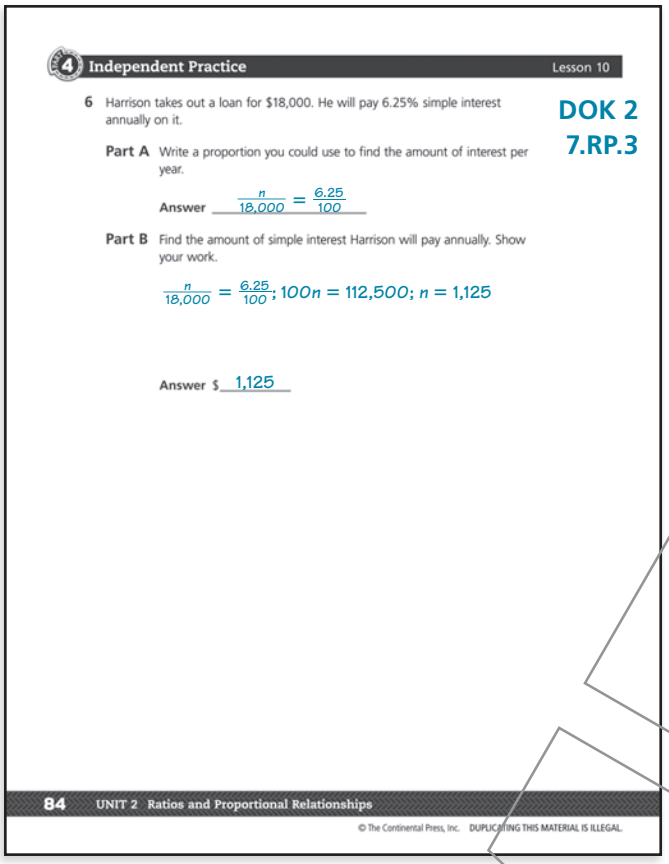

4 Independent Practice Lesson 10

6 Harrison takes out a loan for $18,000. He will pay 6.25% simple interest annually on it.

DOK 2
7.RP.3

Part A Write a proportion you could use to find the amount of interest per year.

Answer $\frac{n}{18,000} = \frac{6.25}{100}$

Part B Find the amount of simple interest Harrison will pay annually. Show your work.

$\frac{n}{18,000} = \frac{6.25}{100}$; $100n = 112,500$; $n = 1,125$

Answer $ 1,125

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6 PART A An amount of simple interest is a percent of a number. The proportion that represents this situation is $\frac{n}{18,000} = \frac{6.25}{100}$.

PART B To solve for the amount of interest, first cross multiply: $112,500 = 100n$. Divide both sides of the equation by 100: $1,125 = n$. The amount of simple interest is $1,125.

Extension Activity

Prepare a spinner divided into 20 equal sections, and label the sections with percentages in increments of 5 from 5% to 100%. Ask students to bring in an advertisement or a printout from a website of an item they would like to buy; it should display an undiscounted price. Let each student spin the arrow to find a discount percent for his or her item. Have the student demonstrate how to set up a proportion with the percent as one ratio and the price in the other, and then find the equation and solve for the amount of discount. Students may want to take this a step further and find the price after the discount.