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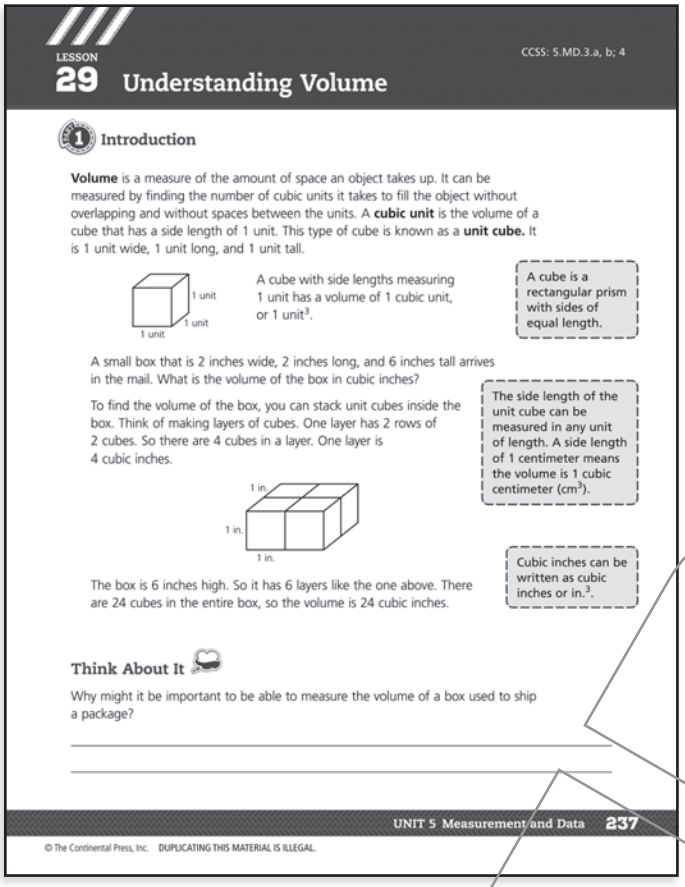
ISBN 978-0-8454-7895-0



LESSON 29 Understanding Volume

CCSS: 5.MD.3.a, b; 4

1 Introduction

Volume is a measure of the amount of space an object takes up. It can be measured by finding the number of cubic units it takes to fill the object without overlapping and without spaces between the units. A **cubic unit** is the volume of a cube that has a side length of 1 unit. This type of cube is known as a **unit cube.** It is 1 unit wide, 1 unit long, and 1 unit tall.

1 unit, 1 unit, 1 unit

A cube with side lengths measuring 1 unit has a volume of 1 cubic unit, or 1 unit^3.

A cube is a rectangular prism with sides of equal length.

A small box that is 2 inches wide, 2 inches long, and 6 inches tall arrives in the mail. What is the volume of the box in cubic inches?

To find the volume of the box, you can stack unit cubes inside the box. Think of making layers of cubes. One layer has 2 rows of 2 cubes. So there are 4 cubes in a layer. One layer is 4 cubic inches.

The side length of the unit cube can be measured in any unit of length. A side length of 1 centimeter means the volume is 1 cubic centimeter (cm^3).

1 in., 1 in., 1 in.

The box is 6 inches high. So it has 6 layers like the one above. There are 24 cubes in the entire box, so the volume is 24 cubic inches.

Cubic inches can be written as cubic inches or in.^{3}.

Think About It

Why might it be important to be able to measure the volume of a box used to ship a package?

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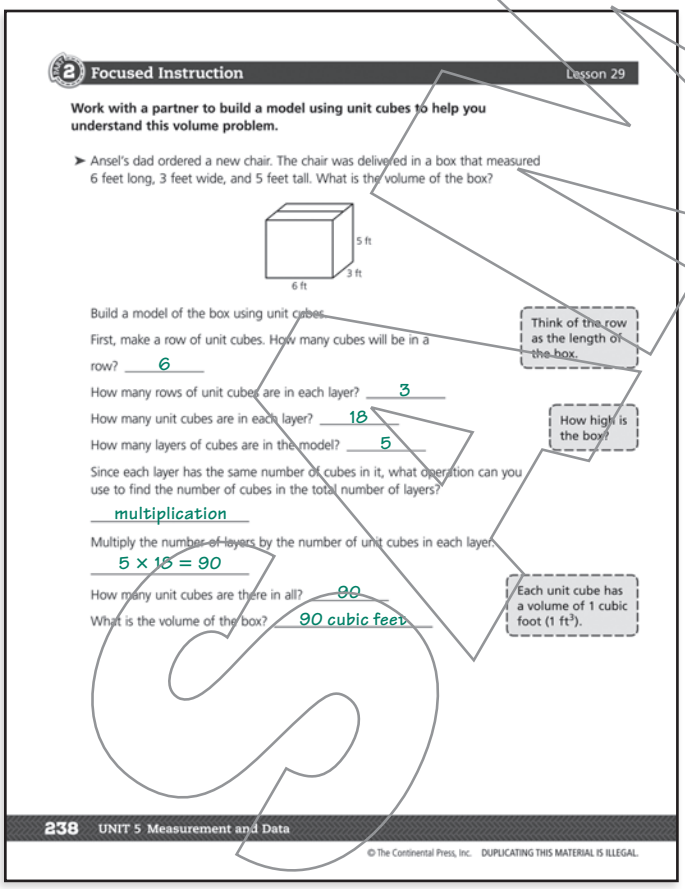

2 Focused Instruction Lesson 29

Work with a partner to build a model using unit cubes to help you understand this volume problem.

➤ Ansel's dad ordered a new chair. The chair was delivered in a box that measured 6 feet long, 3 feet wide, and 5 feet tall. What is the volume of the box?

5 ft, 3 ft, 6 ft

Build a model of the box using unit cubes.

First, make a row of unit cubes. How many cubes will be in a row? 6

Think of the row as the length of the box.

How many rows of unit cubes are in each layer? 3

How many unit cubes are in each layer? 18

How many layers of cubes are in the model? 5

How high is the box?

Since each layer has the same number of cubes in it, what operation can you use to find the number of cubes in the total number of layers?

multiplication

Multiply the number of layers by the number of unit cubes in each layer.

5 × 18 = 90

How many unit cubes are there in all? 90

What is the volume of the box? 90 cubic feet

Each unit cube has a volume of 1 cubic foot (1 ft^3).

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Objective

To understand volume and cubic units

PART 1 Introduction

Review volume as the measurement of space inside a solid object and that volume is measured in cubic units. Use the unit cube diagram to show that a unit cube has length, width, and height. Work through the sample item as a class.

Think About It

Students should recognize that finding volume is important in order to find a box that is the appropriate size for mailing a package. They need to make sure that whatever they are mailing will fit inside a box and that the box is not too large, since that can increase the cost to mail it.

Common Core State Standards

5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- **a.** A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
- **b.** A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft, and improvised units.

Vocabulary

cubic unit: the amount of space inside a cube that measures 1 unit on each edge

unit cube: a cube that measures 1 unit wide, 1 unit long, and 1 unit high

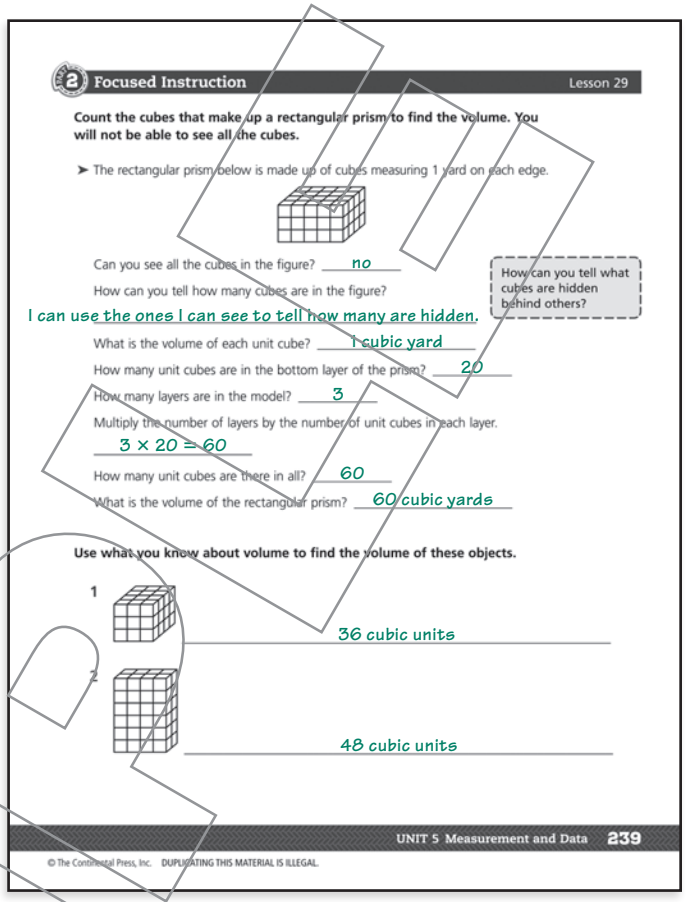
volume: the amount of space inside an object

Part 2 Focused Instruction

First, students work in groups of two to build a model to help them solve the problem. Provide each pair of students with 90 unit cubes in order to create the model. Alternatively, provide each pair of students with 22 unit cubes so they can create one layer of the model as well as the height. Students will work together to understand how the model shows the same thing as the picture. They will use the model to find the volume.

Next, students will understand that they can count some of the cubes in a model and still find the volume. They will work through a series of questions that help them think about the unit cubes they cannot see in the image. They will then find the volume.

Conclude the Focused Instruction section by having students find the volume of two objects.



Focused Instruction Lesson 29

Count the cubes that make up a rectangular prism to find the volume. You will not be able to see all the cubes.

➤ The rectangular prism below is made up of cubes measuring 1 yard on each edge.

Can you see all the cubes in the figure? no

How can you tell how many cubes are in the figure?

I can use the ones I can see to tell how many are hidden.

How can you tell what cubes are hidden behind others?

What is the volume of each unit cube? 1 cubic yard

How many unit cubes are in the bottom layer of the prism? 20

How many layers are in the model? 3

Multiply the number of layers by the number of unit cubes in each layer.

3 × 20 = 60

How many unit cubes are there in all? 60

What is the volume of the rectangular prism? 60 cubic yards

Use what you know about volume to find the volume of these objects.

1 36 cubic units

2 48 cubic units

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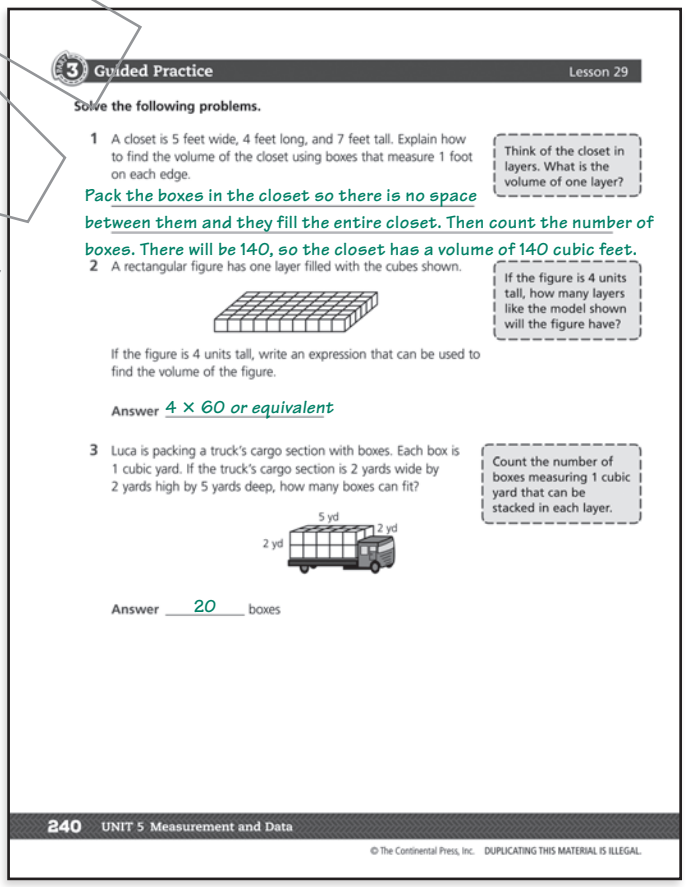
Part 3 Guided Practice

Students should complete the Guided Practice section on their own. Offer assistance as needed, pointing out the reminder and hint boxes along the right side of the page.



Guided Practice Lesson 29

Solve the following problems.

1 A closet is 5 feet wide, 4 feet long, and 7 feet tall. Explain how to find the volume of the closet using boxes that measure 1 foot on each edge.

Think of the closet in layers. What is the volume of one layer?

Pack the boxes in the closet so there is no space between them and they fill the entire closet. Then count the number of boxes. There will be 140, so the closet has a volume of 140 cubic feet.

2 A rectangular figure has one layer filled with the cubes shown.

If the figure is 4 units tall, how many layers like the model shown will the figure have?

If the figure is 4 units tall, write an expression that can be used to find the volume of the figure.

Answer 4 × 60 or equivalent

3 Luca is packing a truck's cargo section with boxes. Each box is 1 cubic yard. If the truck's cargo section is 2 yards wide by 2 yards high by 5 yards deep, how many boxes can fit?

Count the number of boxes measuring 1 cubic yard that can be stacked in each layer.

5 yd, 2 yd, 2 yd

Answer 20 boxes

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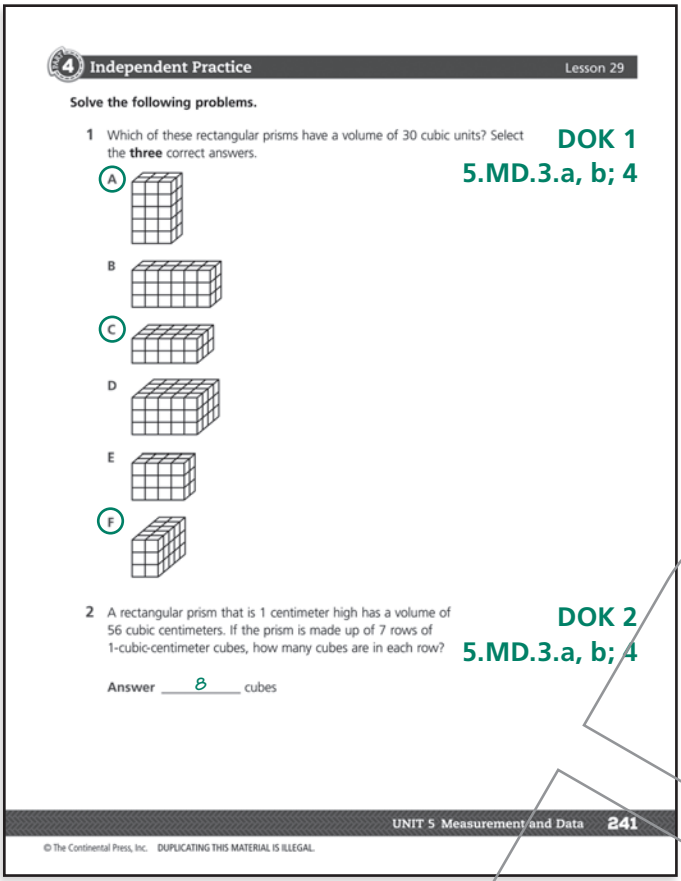
Connections to Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.



Independent Practice Lesson 29

Solve the following problems.

1 Which of these rectangular prisms have a volume of 30 cubic units? Select the **three** correct answers. DOK 1 5.MD.3.a, b; 4

A

B

C

D

E

F

2 A rectangular prism that is 1 centimeter high has a volume of 56 cubic centimeters. If the prism is made up of 7 rows of 1-cubic-centimeter cubes, how many cubes are in each row? DOK 2 5.MD.3.a, b; 4

Answer 8 cubes

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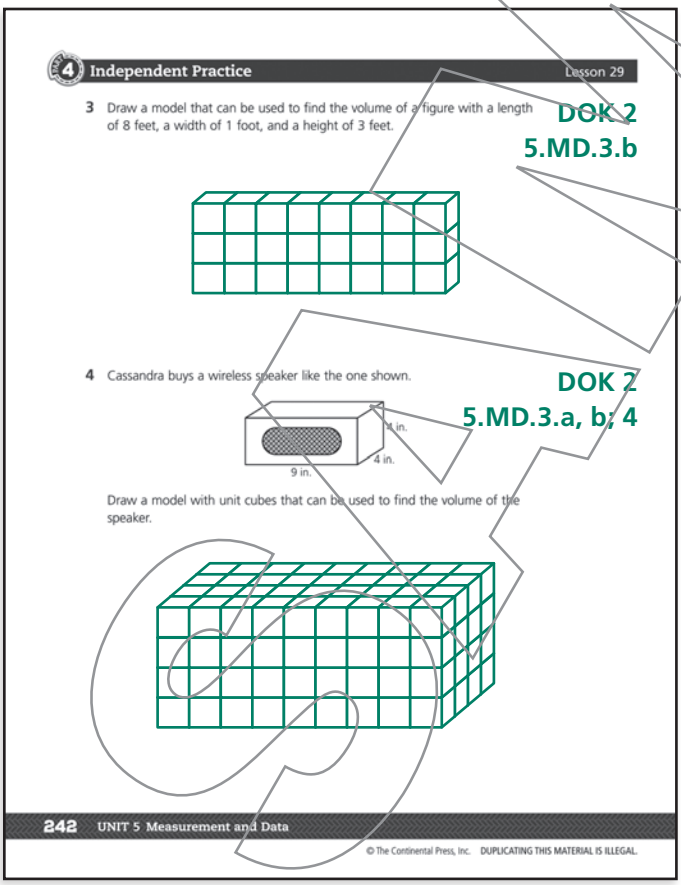

Independent Practice Lesson 29

3 Draw a model that can be used to find the volume of a figure with a length of 8 feet, a width of 1 foot, and a height of 3 feet. DOK 2 5.MD.3.b

4 Cassandra buys a wireless speaker like the one shown. DOK 2 5.MD.3.a, b; 4

Draw a model with unit cubes that can be used to find the volume of the speaker.

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Part 4 Independent Practice Answer Rationales

1 Count the unit cubes that make up each figure. Choices A, C, and F are made up of 30 unit cubes, so they have volumes of 30 cubic units. Choice B has a volume of 36 cubic units. Choice D has a volume of 60 cubic units. Choice E has a volume of 24 cubic units.

2 A prism with a height of 1 unit will have to have all the cubic units in 1 layer, so a base with 56 cubic centimeters would have a length of 7 and a width of 8 because $8 \times 7 = 56$.

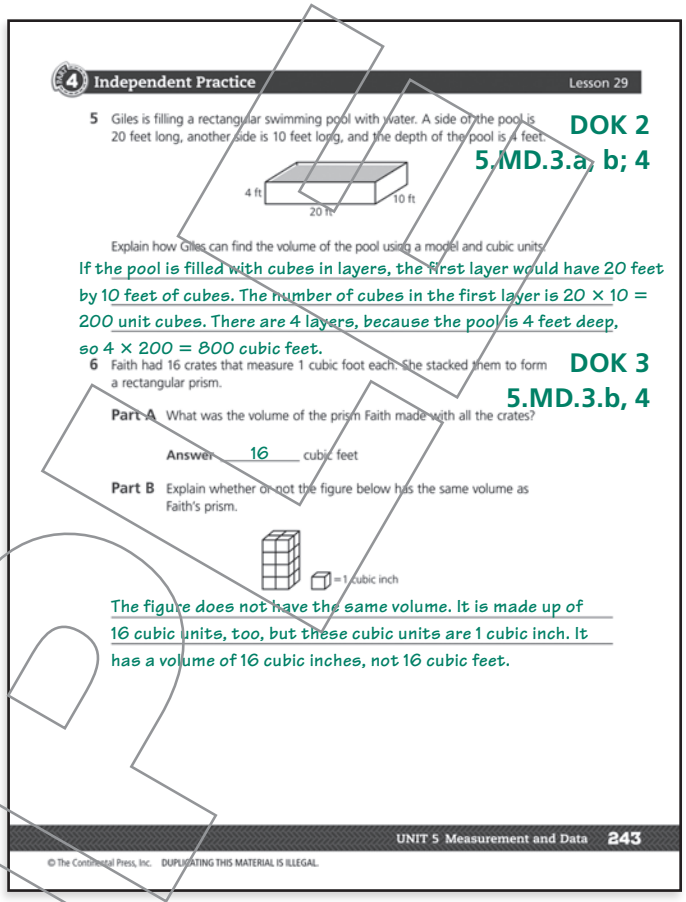
3 A figure with a length of 8, a width of 1, and height of 3 can be divided into 1 row of 8 cubes with a total of 3 rows.

4 The speaker is 9 inches long, so the model has a length of 9 unit cubes. The speaker is 4 inches wide and 4 inches high, so the model has a width of 4 unit cubes and a height of 4 unit cubes. Each layer has $9 \times 4 = 36$ unit cubes. There are 4 layers, so the volume is the sum of the number of cubes in each layer: $36 + 36 + 36 + 36 = 144$ cubic units.

5 The pool is a rectangular prism that can be filled with cubic feet. If the bottom of the pool measures 10 feet by 20 feet, then the bottom of the pool would have $10 \times 20 = 200$ cubic units. The pool is 4 feet deep, so there would be 4 layers of 200 units: $4 \times 200 = 800$ cubic feet.

6 PART A There are 16 crates in the stack. Each crate has a volume of 1 cubic foot, so the total volume of the stack is 16 cubic feet.

PART B To have the same volume, the figures must be made up of the same cubes with the same cubic unit. Faith's stack is made up of crates that are 1 cubic foot each. The second figure is made up of cubes that are 1 cubic inch each. So Faith's figure is 16 cubic feet and the second figure is 16 cubic inches. These are not the same volume.



4 Independent Practice Lesson 29

5 Giles is filling a rectangular swimming pool with water. A side of the pool is 20 feet long, another side is 10 feet long, and the depth of the pool is 4 feet. DOK 2 5.MD.3.a, b; 4

Explain how Giles can find the volume of the pool using a model and cubic units.

If the pool is filled with cubes in layers, the first layer would have 20 feet by 10 feet of cubes. The number of cubes in the first layer is 20 × 10 = 200 unit cubes. There are 4 layers, because the pool is 4 feet deep, so 4 × 200 = 800 cubic feet.

6 Faith had 16 crates that measure 1 cubic foot each. She stacked them to form a rectangular prism. DOK 3 5.MD.3.b, 4

Part A What was the volume of the prism Faith made with all the crates?

Answer 16 cubic feet

Part B Explain whether or not the figure below has the same volume as Faith's prism.

The figure does not have the same volume. It is made up of 16 cubic units, too, but these cubic units are 1 cubic inch. It has a volume of 16 cubic inches, not 16 cubic feet.

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Extension Activity

Provide groups of students with unit cubes. Have them construct models with specified volumes, such as 20 cubic units, 36 cubic units, and 100 cubic units. Sometimes there may be more than one way to make a model with a specified volume. Have students think about different ways to make figures with the same volume.

