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LESSON 4

Solving Systems of Equations by Substitution

Another way to solve systems of equations algebraically is the substitution method. In **substitution,** one equation is written in terms of a single variable. Then the expression equal to that variable is substituted into the other equation.

What is the solution to this system of equations?

$$\begin{cases} x - 3y = 2 \\ 2x - 5y = 2 \end{cases}$$

Set the first equation equal to *x*.

$$x - 3y = 2 \quad \rightarrow \quad x = 3y + 2$$

Substitute $3y + 2$ for *x* in the second equation.

$$2(3y + 2) - 5y = 2$$

Solve the equation for *y*.

$$2(3y + 2) - 5y = 2 \quad \rightarrow \quad \begin{aligned} 6y + 4 - 5y &= 2 \\ y + 4 &= 2 \\ y &= \text{-}2 \end{aligned}$$

Substitute $y = \text{-}2$ into either equation in the system. Solve for *x*.

$$\begin{aligned} x - 3(\text{-}2) &= 2 \\ x + 6 &= 2 \\ x &= \text{-}4 \end{aligned}$$

Check that $x = \text{-}4$ and $y = \text{-}2$, or (-4, -2), is the solution. Substitute both values into each equation. See if the equations are both true.

$$\begin{aligned} x - 3y &= 2 \\ \text{-}4 - 3(\text{-}2) &= 2 \\ \text{-}4 + 6 &= 2 \\ 2 &= 2 \quad \text{true} \end{aligned} \qquad \begin{aligned} 2x - 5y &= 2 \\ 2(\text{-}4) - 5(\text{-}2) &= 2 \\ \text{-}8 + 10 &= 2 \\ 2 &= 2 \quad \text{true} \end{aligned}$$

Both equations are true, so (-4, -2) is the solution.

It doesn't matter which equation or which variable, *x* or *y*, you choose to solve for. You should still be able to find the solution.

It is easier to solve for a variable that has 1 as a coefficient.

$$1x = x$$

Remember to fully distribute the factor into each term within the parentheses.

$$\begin{aligned} 4(2x + 7) &= 4(2x) + 4(7) \\ &= 8x + 28 \end{aligned}$$

Remember that some systems of equations can have no solution or infinitely many solutions.

For systems with no solution, the substituted equation would result in $a = b$, which is false.

For systems with infinitely many solutions, the substituted equation would result in $a = a$, which is always true.

GUIDED PRACTICE

Read and solve each problem.

1 Look at this system of equations.

$$\begin{cases} y = x + 2 \\ y = 5x - 2 \end{cases}$$

What is the solution to this system?

A (1, 3)

B (-1, 3)

C (3, 1)

D (3, -1)

Set the value of *y* in one equation equal to the value of *y* in the other equation.

2 Which best describes the solution to $\begin{cases} 6x + 3y = 4 \\ 2x + y = 2 \end{cases}$?

A (0, 0)

B (4, 2)

C no solution

D infinitely many solutions

The second equation has a *y*-term with a coefficient of 1, so solve the second equation for *y*.

3 What is the solution to this system of equations? Show your work.

$$\begin{cases} x + y = 5 \\ x = y + 5 \end{cases}$$

Which expression is equal to a variable in the other equation?

Answer ____________

TEST YOURSELF

Read and solve each problem.

1 Is (-5, 3) a solution to this system of equations?

$$\begin{cases} x + y = -2 \\ -2x + y = 13 \end{cases}$$

A Yes, because it is a solution to both equations.

B No, because it is not a solution to the first equation.

C No, because it is not a solution to the second equation.

D No, because it is not a solution to either equation.

2 Look at this system of equations.

$$\begin{cases} x + 4y = 3 \\ y = x - 8 \end{cases}$$

What is the solution to this system?

A (-1, -1) C (1, -1)

B (-1, 7) D (7, -1)

3 What is the solution to this system of equations?

$$\begin{cases} x = 2y - 1 \\ x = y + 3 \end{cases}$$

A (2, 5) C (5, 2)

B (4, 7) D (7, 4)

4 What is the solution to this system of equations?

$$\begin{cases} 2x = -6y + 4 \\ 3x + 9y = 6 \end{cases}$$

A (0, 0)

B (2, 0)

C no solution

D infinitely many solutions

5 What is the solution to this system of equations?

$$\begin{cases} x = -3y - 2 \\ y = x + 2 \end{cases}$$

A (2, 0) C (0, 2)

B (-2, 0) D (0, -2)

6 What is the solution to this system of equations? Show your work.

$$\begin{cases} -x - 2y = -3 \\ 3x + 3y = 0 \end{cases}$$

Answer ____________

7 Look at this system of equations.

$$\begin{cases} x + y = -2 \\ 2x + y = 3 \end{cases}$$

Part A What is the solution to this system of equations?

Answer ____________

Part B Explain how you found your answer.

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8 Lamont wrote this system of equations.

$$\begin{cases} x = 2y + 2 \\ y = x + 1 \end{cases}$$

Part A Lamont started to solve this system of equations using the equation $2y + 2 = x + 1$. Will this equation help him get the solution to this system of equations? Explain how you know.

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Part B What is the solution? Show your work.

Answer ____________