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NYS NEXT GENERATION MATHEMATICS LEARNING STANDARDS

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

8.EE.8.a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. Recognize when the system has one solution, no solution, or infinitely many solutions.

8.EE.8.b Solve systems of two linear equations in two variables with integer coefficients: graphically, numerically using a table, and algebraically. Solve simple cases by inspection.

Introduction

The lesson reviews how to solve systems of linear equations by the substitution method. Read or have a volunteer read through the lesson and discuss the examples with the class. Guide students to understand that the object is to replace a variable with its value in terms of the other variable. Emphasize that the result is an equation in one variable, which they can solve for the value of that variable.

Guided Practice

The guided practice page provides sample multiple-choice and constructed answer problems for the students to complete on their own. Each item is accompanied by a hint or reminder that guides the student's thinking about how to solve the problem. Offer assistance as needed. When students have completed the items, review the answers and solution processes as a class.

LESSON 4

Solving Systems of Equations by Substitution

Another way to solve systems of equations algebraically is the substitution method. In **substitution,** one equation is written in terms of a single variable. Then the expression equal to that variable is substituted into the other equation.

What is the solution to this system of equations?

$$\begin{cases} x - 3y = 2 \\ 2x - 5y = 2 \end{cases}$$

Set the first equation equal to *x*.

$$x - 3y = 2 \quad \rightarrow \quad x = 3y + 2$$

Substitute $3y + 2$ for *x* in the second equation.

$$2(3y + 2) - 5y = 2$$

Solve the equation for *y*.

$$2(3y + 2) - 5y = 2 \quad \rightarrow \quad 6y + 4 - 5y = 2$$
$$y + 4 = 2$$
$$y = -2$$

Substitute $y = -2$ into either equation in the system. Solve for *x*.

$$x - 3(-2) = 2$$
$$x + 6 = 2$$
$$x = -4$$

Check that $x = -4$ and $y = -2$, or (-4, -2), is the solution. Substitute both values into each equation. See if the equations are both true.

$x - 3y = 2$	$2x - 5y = 2$
$-4 - 3(-2) = 2$	$2(-4) - 5(-2) = 2$
$-4 + 6 = 2$	$-8 + 10 = 2$
$2 = 2$ true	$2 = 2$ true

Both equations are true, so (-4, -2) is the solution.

It doesn't matter which equation or which variable, *x* or *y*, you choose to solve for. You should still be able to find the solution.

It is easier to solve for a variable that has 1 as a coefficient.

$1x = x$

Remember to fully distribute the factor into each term within the parentheses.

$4(2x + 7) = 4(2x) + 4(7)$
$= 8x + 28$

Remember that some systems of equations can have no solution or infinitely many solutions.

For systems with no solution, the substituted equation would result in $a = b$, which is false.

For systems with infinitely many solutions, the substituted equation would result in $a = a$, which is always true.

GUIDED PRACTICE

Read and solve each problem.

1 Look at this system of equations.

$$\begin{cases} y = x + 2 \\ y = 5x - 2 \end{cases}$$

What is the solution to this system?

Ⓐ (1, 3)
B (-1, 3)
C (3, 1)
D (3, -1)

Set the value of *y* in one equation equal to the value of *y* in the other equation.

2 Which best describes the solution to $\begin{cases} 6x + 3y = 4 \\ 2x + y = 2 \end{cases}$?

A (0, 0)
B (4, 2)
Ⓒ no solution
D infinitely many solutions

The second equation has a *y*-term with a coefficient of 1, so solve the second equation for *y*.

3 What is the solution to this system of equations? Show your work.

$$\begin{cases} x + y = 5 \\ x = y + 5 \end{cases}$$

$$y + 5 + y = 5$$
$$2y + 5 = 5$$
$$2y = 0$$
$$y = 0$$

$$x + 0 = 5$$
$$x = 5$$

Answer (5, 0)

Which expression is equal to a variable in the other equation?

Answer Rationales

1. A solution is correct if it makes both equations in the system true. Substitute (–5, 3) into each equation: $-5 + 3 = -2 \rightarrow -2 = -2$ and $-2(-5) + 3 = 13 \rightarrow 10 + 3 = 13 \rightarrow 13 = 13$. The solution makes both equations true. Choice A is correct. ***(8.EE.8.b)***

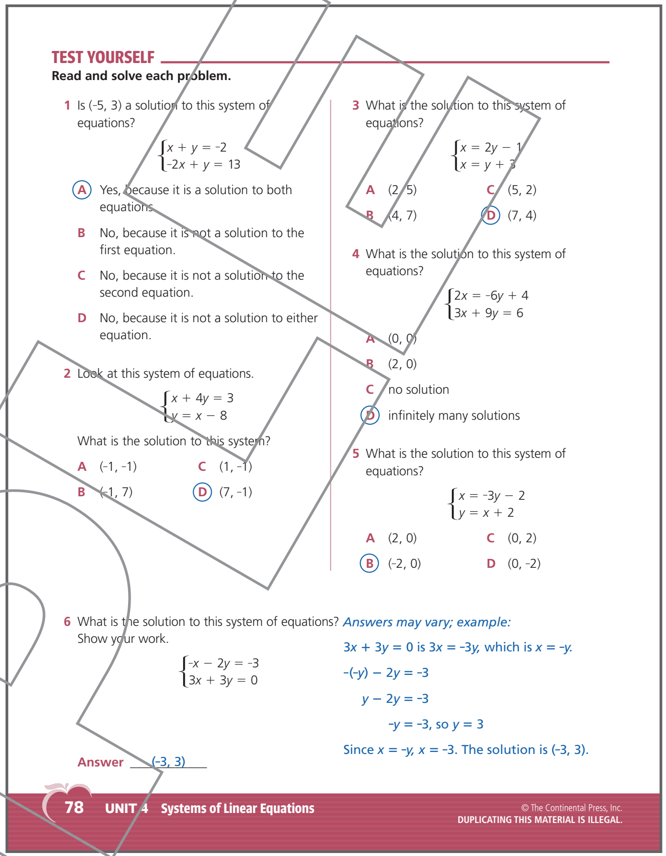
2. The second equation is in terms of y, so substitute the expression that equals y for y in the first equation: $x + 4y = 3 \rightarrow x + 4(x - 8) = 3 \rightarrow x + 4x - 32 = 3 \rightarrow 5x = 35 \rightarrow x = 7$. Substitute the value of x into the second equation to find the value of y: $y = 7 - 8 \rightarrow y = -1$. The solution to the system of equations is (7, –1). Choice D is correct. ***(8.EE.8.b)***

3. Substitute the expression equal to x for x in the first equation: $y + 3 = 2y - 1$. Simplify to $3 = y - 1 \rightarrow 4 = y$. Use this value of y in the second equation to find the value of x: $x = 4 + 3 \rightarrow x = 7$. The solution is (7, 4). Choice D is correct. ***(8.EE.8.b)***

4. First, rewrite the first equation in terms of x: $2x \div 2 = (-6y + 4) \div 2 \rightarrow x = -3y + 2$. Substitute the expression equal to x for x in the second equation: $3(-3y + 2) + 9y = 6 \rightarrow -9y + 6 + 9y = 6$. This simplifies to $6 = 6$. So any value of x and y will make the system true, and the system has infinitely many solutions. Choice D is correct. ***(8.EE.8.a, b)***

5. Since $y = x + 2$, substitute $x + 2$ for y in the first equation: $x = -3(x + 2) - 2 \rightarrow x = -3x - 6 - 2 \rightarrow 4x = -8 \rightarrow x = -2$. Use this value for x in the second equation: $y = -2 + 2 = 0$. The solution is (–2, 0). Choice B is correct. ***(8.EE.8.b)***

6. Since $3x + 3y = 0$, then $3x = -3y$, or $x = -y$. Substitute $-y$ for x in the first equation: $-(-y) - 2y = -3 \rightarrow y - 2y = -3 \rightarrow -y = -3$, so $y = 3$. The value of x is $-y$, so x is -3. The solution is (–3, 3). ***(8.EE.8.b)***



TEST YOURSELF

Read and solve each problem.

1 Is (-5, 3) a solution to this system of equations?

$$\begin{cases} x + y = -2 \\ -2x + y = 13 \end{cases}$$

(A) Yes, because it is a solution to both equations.
B No, because it is not a solution to the first equation.
C No, because it is not a solution to the second equation.
D No, because it is not a solution to either equation.

2 Look at this system of equations.

$$\begin{cases} x + 4y = 3 \\ y = x - 8 \end{cases}$$

What is the solution to this system?

A (-1, -1) C (1, -1)
B (-1, 7) (D) (7, -1)

3 What is the solution to this system of equations?

$$\begin{cases} x = 2y - 1 \\ x = y + 3 \end{cases}$$

A (2, 5) C (5, 2)
B (4, 7) (D) (7, 4)

4 What is the solution to this system of equations?

$$\begin{cases} 2x = -6y + 4 \\ 3x + 9y = 6 \end{cases}$$

A (0, 0)
B (2, 0)
C no solution
(D) infinitely many solutions

5 What is the solution to this system of equations?

$$\begin{cases} x = -3y - 2 \\ y = x + 2 \end{cases}$$

A (2, 0) C (0, 2)
(B) (-2, 0) D (0, -2)

6 What is the solution to this system of equations? Show your work.

$$\begin{cases} -x - 2y = -3 \\ 3x + 3y = 0 \end{cases}$$

Answers may vary; example:
$3x + 3y = 0$ is $3x = -3y$, which is $x = -y$.
$-(-y) - 2y = -3$
$y - 2y = -3$
$-y = -3$, so $y = 3$
Since $x = -y$, $x = -3$. The solution is (-3, 3).

Answer (-3, 3)

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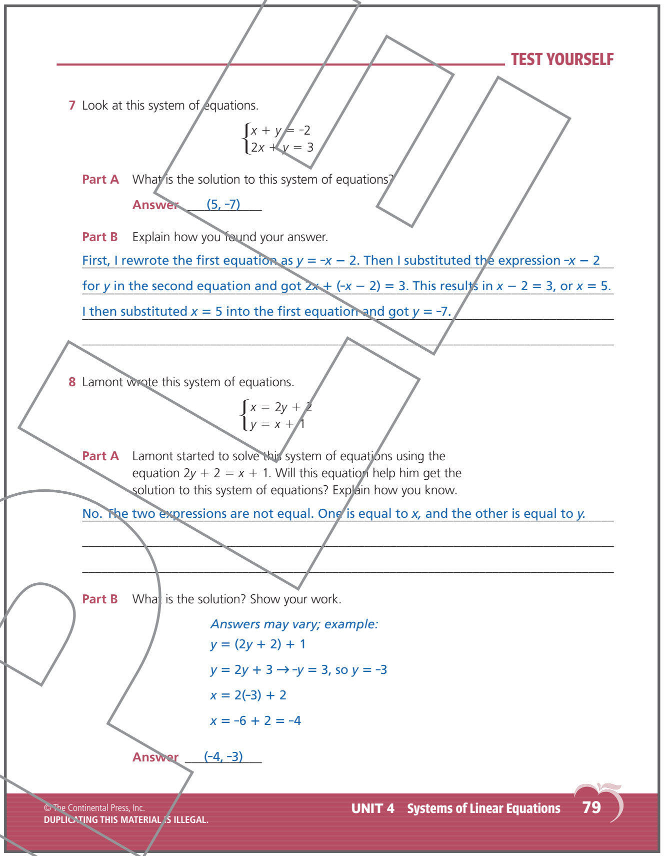
7. Parts A and B The first equation can be rewritten in terms of y as $y = -x - 2$. Substitute this expression for y in the second equation: $2x + (-x - 2) = 3 \rightarrow 2x - x - 2 = 3 \rightarrow x - 2 = 3 \rightarrow x = 5$. Substitute the value of x in the first equation to find y: $5 + y = -2 \rightarrow y = -7$. The solution is $(5, -7)$. ***(8.EE.8.b)***

8. Part A The expression $2y + 2$ is equal to x, not y; Lamont cannot solve for the value of one variable if the equation contains two variables. ***(8.EE.8.b)***

Part B Substitute the expression that equals x for x in the second equation: $y = 2y + 2 + 1 \rightarrow y = 2y + 3 \rightarrow -y = 3$, so $y = -3$. Substitute -3 for y in the second equation: $-3 = x + 1 \rightarrow -4 = x$. So the solution is $(-4, -3)$. ***(8.EE.8.b)***


TEST YOURSELF

7 Look at this system of equations.

$$\begin{cases} x + y = -2 \\ 2x + y = 3 \end{cases}$$

Part A What is the solution to this system of equations?

Answer $(5, -7)$

Part B Explain how you found your answer.

First, I rewrote the first equation as $y = -x - 2$. Then I substituted the expression $-x - 2$ for y in the second equation and got $2x + (-x - 2) = 3$. This results in $x - 2 = 3$, or $x = 5$. I then substituted $x = 5$ into the first equation and got $y = -7$.

8 Lamont wrote this system of equations.

$$\begin{cases} x = 2y + 2 \\ y = x + 1 \end{cases}$$

Part A Lamont started to solve this system of equations using the equation $2y + 2 = x + 1$. Will this equation help him get the solution to this system of equations? Explain how you know.

No. The two expressions are not equal. One is equal to x, and the other is equal to y.

Part B What is the solution? Show your work.

Answers may vary; example:

$y = (2y + 2) + 1$

$y = 2y + 3 \rightarrow -y = 3$, so $y = -3$

$x = 2(-3) + 2$

$x = -6 + 2 = -4$

Answer $(-4, -3)$

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CONNECTING TO MATHEMATICAL CONTENT

Grade-span connections:

7.EE.2 → 8.EE.8 → AI-A.REI.6.a, AI-A.REI.7.a

Grade-level connections:

8.F.3 (interpreting functions)

8.F.4 (constructing functions)

CONNECTING TO MATHEMATICAL PRACTICES

MP2: Reason abstractly and quantitatively.

MP3: Construct viable arguments and critique the reasoning of others.

MP6: Attend to precision.

MP7: Look for and make use of structure.