TABLE of CONTENTS

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NYS NEXT GENERATION MATHEMATICS LEARNING STANDARDS

6.SP.7 Approximate the probability of a simple event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probabilities.

6.SP.8 Develop a probability model and use it to find probabilities of simple events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

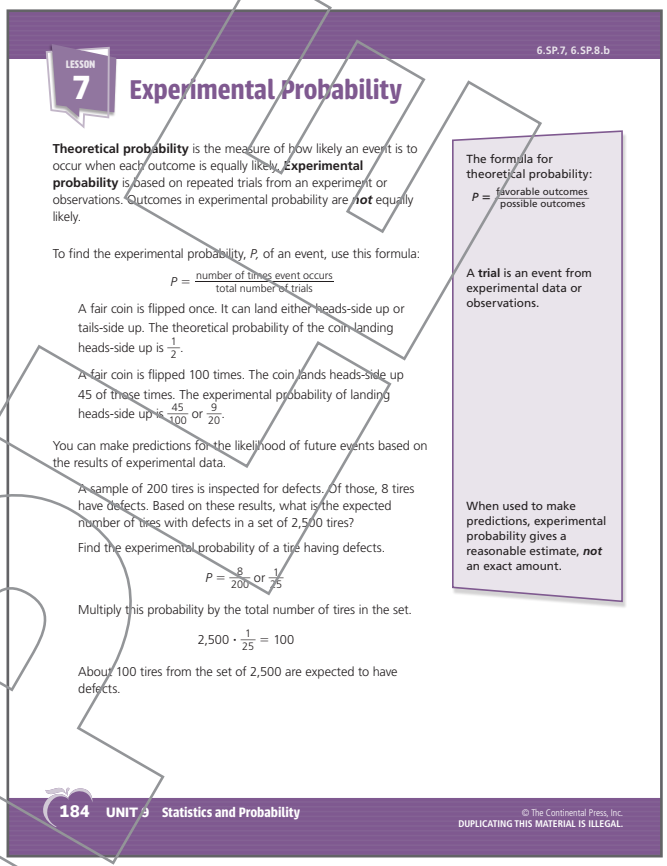
6.SP.8.b Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

Introduction

The lesson reviews experimental probability. Read or have a volunteer read through the lesson and discuss the examples with the class. Discuss the difference between theoretical and experimental probability and present additional examples to the class as needed.

Guided Practice

The guided practice page provides sample multiple-choice and constructed answer problems for the students to complete on their own. Each item is accompanied by a hint or reminder that guides the student's thinking about how to solve the problem. Offer assistance as needed. When students have completed the items, review the answers and solution processes as a class.



6.SP.7, 6.SP.8.b

LESSON 7

Experimental Probability

Theoretical probability is the measure of how likely an event is to occur when each outcome is equally likely. **Experimental probability** is based on repeated trials from an experiment or observations. Outcomes in experimental probability are ***not*** equally likely.

To find the experimental probability, P, of an event, use this formula:

$$P = \frac{\text{number of times event occurs}}{\text{total number of trials}}$$

A fair coin is flipped once. It can land either heads-side up or tails-side up. The theoretical probability of the coin landing heads-side up is $\frac{1}{2}$.

A fair coin is flipped 100 times. The coin lands heads-side up 45 of those times. The experimental probability of landing heads-side up is $\frac{45}{100}$ or $\frac{9}{20}$.

You can make predictions for the likelihood of future events based on the results of experimental data.

A sample of 200 tires is inspected for defects. Of those, 8 tires have defects. Based on these results, what is the expected number of tires with defects in a set of 2,500 tires?

Find the experimental probability of a tire having defects.

$$P = \frac{8}{200} \text{ or } \frac{1}{25}$$

Multiply this probability by the total number of tires in the set.

$$2{,}500 \cdot \frac{1}{25} = 100$$

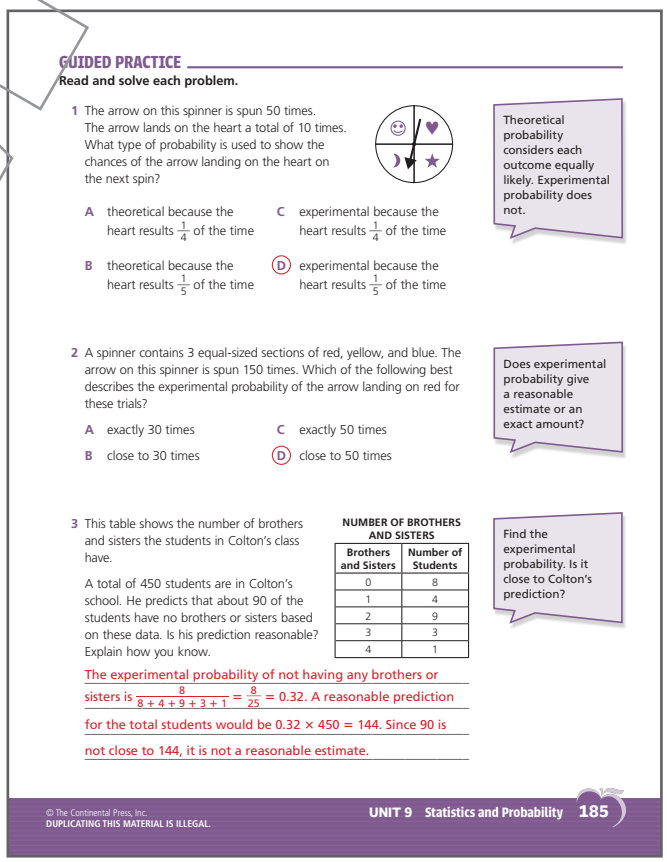
About 100 tires from the set of 2,500 are expected to have defects.

The formula for theoretical probability: $P = \frac{\text{favorable outcomes}}{\text{possible outcomes}}$

A **trial** is an event from experimental data or observations.

When used to make predictions, experimental probability gives a reasonable estimate, ***not*** an exact amount.

184 UNIT 9 Statistics and Probability



GUIDED PRACTICE

Read and solve each problem.

1 The arrow on this spinner is spun 50 times. The arrow lands on the heart a total of 10 times. What type of probability is used to show the chances of the arrow landing on the heart on the next spin?

A theoretical because the heart results $\frac{1}{4}$ of the time

B theoretical because the heart results $\frac{1}{5}$ of the time

C experimental because the heart results $\frac{1}{4}$ of the time

(D) experimental because the heart results $\frac{1}{5}$ of the time

Theoretical probability considers each outcome equally likely. Experimental probability does not.

2 A spinner contains 3 equal-sized sections of red, yellow, and blue. The arrow on this spinner is spun 150 times. Which of the following best describes the experimental probability of the arrow landing on red for these trials?

A exactly 30 times

B close to 30 times

C exactly 50 times

(D) close to 50 times

Does experimental probability give a reasonable estimate or an exact amount?

3 This table shows the number of brothers and sisters the students in Colton's class have.

NUMBER OF BROTHERS AND SISTERS

Brothers and Sisters	Number of Students
0	8
1	4
2	9
3	3
4	1

A total of 450 students are in Colton's school. He predicts that about 90 of the students have no brothers or sisters based on these data. Is his prediction reasonable? Explain how you know.

The experimental probability of not having any brothers or sisters is $\frac{8}{8+4+9+3+1} = \frac{8}{25} = 0.32$. A reasonable prediction for the total students would be $0.32 \times 450 = 144$. Since 90 is not close to 144, it is not a reasonable estimate.

Find the experimental probability. Is it close to Colton's prediction?

UNIT 9 Statistics and Probability 185

Answer Rationales

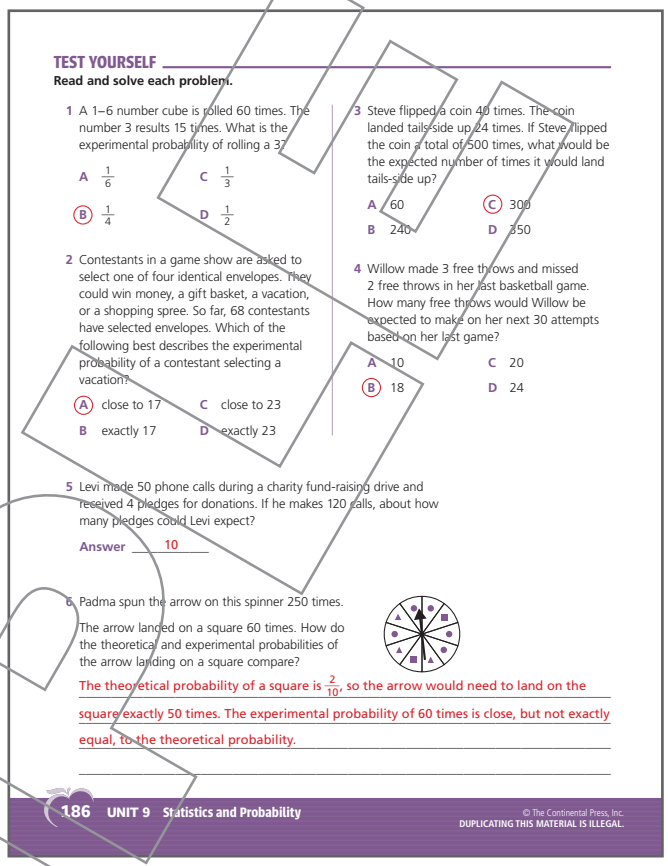
1. The experimental probability is based on the repeated trials. There were 60 rolls, and the number 3 resulted 15 times. So the experimental probability is $\frac{15}{60}$, which reduces to $\frac{1}{4}$. Choice B is correct. *(6.SP.8.b)*

2. The experimental probability can be used to make a reasonable estimate. Contestants have a $\frac{1}{4}$ chance of selecting the envelope that gives them a vacation. Use this to make a prediction. There are 68 contestants. Multiply by the experimental probability: $68 \times \frac{1}{4} = 17$. This is an estimate and not an exact amount, so the experimental probability predicts that about 17 of the contestants will win a free vacation. Choice A is correct. *(6.SP.7)*

3. Based on Steve's experiments, the probability of the coin landing on tails is $\frac{24}{40} = \frac{3}{5}$ or 0.6. Multiply this probability by the number of flips to find the expected number of times the coin will land on tails: $500 \times 0.6 = 300$. Choice C is correct. *(6.SP.8.b)*

4. In her last game, Willow had 5 free throw attempts and made 3. So the probability of her making a free throw based on this is $\frac{3}{5}$, or 0.6. Multiply to find the experimental probability for her next 30 attempts: $30 \times \frac{3}{5} = 18$. She will likely make about 18 of the free throws. Choice B is correct. *(6.SP.8.b)*

5. Find the experimental probability by using the probability formula with the number of favorable results, 4, and the number of trials, 50: $\frac{4}{50} = \frac{2}{25} = 0.08$. Use the number of calls that Levi plans to make and multiply it to make a prediction of the number of pledges he can expect: $120 \times \frac{2}{25} = 9.6$. Round the decimal up. 10. He can expect about 10 pledges in 120 calls. *(6.SP.8.b)*



TEST YOURSELF

Read and solve each problem.

1 A 1–6 number cube is rolled 60 times. The number 3 results 15 times. What is the experimental probability of rolling a 3?

A $\frac{1}{6}$ C $\frac{1}{3}$

(B) $\frac{1}{4}$ D $\frac{1}{2}$

2 Contestants in a game show are asked to select one of four identical envelopes. They could win money, a gift basket, a vacation, or a shopping spree. So far, 68 contestants have selected envelopes. Which of the following best describes the experimental probability of a contestant selecting a vacation?

(A) close to 17 C close to 23

B exactly 17 D exactly 23

3 Steve flipped a coin 40 times. The coin landed tails-side up 24 times. If Steve flipped the coin a total of 500 times, what would be the expected number of times it would land tails-side up?

A 60 (C) 300

B 240 D 350

4 Willow made 3 free throws and missed 2 free throws in her last basketball game. How many free throws would Willow be expected to make on her next 30 attempts based on her last game?

A 10 C 20

(B) 18 D 24

5 Levi made 50 phone calls during a charity fund-raising drive and received 4 pledges for donations. If he makes 120 calls, about how many pledges could Levi expect?

Answer 10

6 Padma spun the arrow on this spinner 250 times. The arrow landed on a square 60 times. How do the theoretical and experimental probabilities of the arrow landing on a square compare?

The theoretical probability of a square is $\frac{2}{10}$, so the arrow would need to land on the square exactly 50 times. The experimental probability of 60 times is close, but not exactly equal, to the theoretical probability.

186 UNIT 9 Statistics and Probability

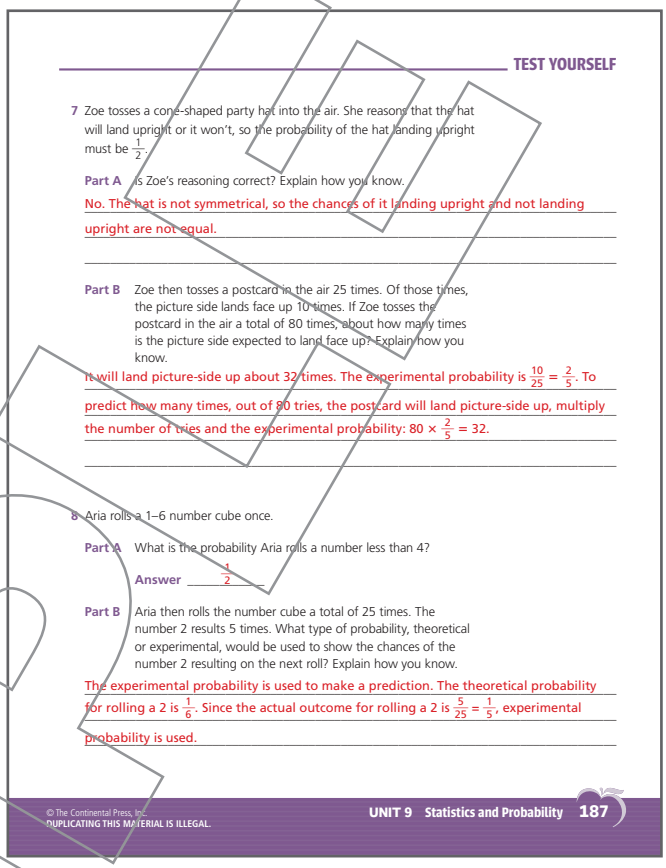
6. The theoretical probability is found by dividing the number of favorable outcomes by the number of possible outcomes. In this case, there are 10 possible outcomes and 2 favorable outcomes, so the theoretical probability is $\frac{2}{10}$ or $\frac{1}{5}$. Multiply this by the number of spins: $250 \times \frac{1}{5} = 50$. Theoretical probability states that the arrow will land on a square 50 times out of 250 spins. After Padma's experiment, the arrow had landed on a square 60 times, which gives an experimental probability of $\frac{60}{250}$. The experimental probability, 60, is close to the theoretical probability, 50, but not the same. *(6.SP.8.b)*

7. Part A Picture a cone-shaped party hat. It is not a symmetrical figure. When tossed in the air, the chance of it landing upright or not is not even. So Zoe's reasoning is incorrect. *(6.SP.7)*

Part B The experimental probability is found by writing the number of times an event occurred over the number of trials: $\frac{10}{25} = \frac{2}{5}$. Use this to make a prediction of the number of times the postcard will land picture-side up in 80 tosses: $80 \times \frac{2}{5} = 32$. It is likely to land picture-side up about 32 times. *(6.SP.7, 8.b)*

8. Part A There are 6 possible results. There are three numbers less than 4: 1, 2, and 3. So the number of favorable outcomes is 3. The probability of Aria rolling a number less than 4 is $\frac{3}{6} = \frac{1}{2}$. *(6.SP.7)*

Part B Use experimental probability to predict the chance of an event occurring. The theoretical probability of Aria rolling a 2 is $\frac{1}{6}$. But in the actual experiment, the probability was $\frac{5}{25} = \frac{1}{5}$. This is a more accurate way to make a prediction. *(6.SP.7)*



TEST YOURSELF

7 Zoe tosses a cone-shaped party hat into the air. She reasons that the hat will land upright or it won't, so the probability of the hat landing upright must be $\frac{1}{2}$.

Part A Is Zoe's reasoning correct? Explain how you know.

No. The hat is not symmetrical, so the chances of it landing upright and not landing upright are not equal.

Part B Zoe then tosses a postcard in the air 25 times. Of those times, the picture side lands face up 10 times. If Zoe tosses the postcard in the air a total of 80 times, about how many times is the picture side expected to land face up? Explain how you know.

It will land picture-side up about 32 times. The experimental probability is $\frac{10}{25} = \frac{2}{5}$. To predict how many times, out of 80 tries, the postcard will land picture-side up, multiply the number of tries and the experimental probability: $80 \times \frac{2}{5} = 32$.

8 Aria rolls a 1–6 number cube once.

Part A What is the probability Aria rolls a number less than 4?

Answer $\frac{1}{2}$

Part B Aria then rolls the number cube a total of 25 times. The number 2 results 5 times. What type of probability, theoretical or experimental, would be used to show the chances of the number 2 resulting on the next roll? Explain how you know.

The experimental probability is used to make a prediction. The theoretical probability for rolling a 2 is $\frac{1}{6}$. Since the actual outcome for rolling a 2 is $\frac{5}{25} = \frac{1}{5}$, experimental probability is used.

CONNECTING TO MATHEMATICAL CONTENT

Grade-span connections:
6.SP.8 → 7.SP.8

Grade-level connections:
6.RP.1–3 (understanding and using ratio and proportional reasoning)

CONNECTING TO MATHEMATICAL PRACTICES

MP1: Make sense of problems and persevere in solving them.

MP7: Look for and make use of structure.

MP8: Look for and express regularity in repeated reasoning.