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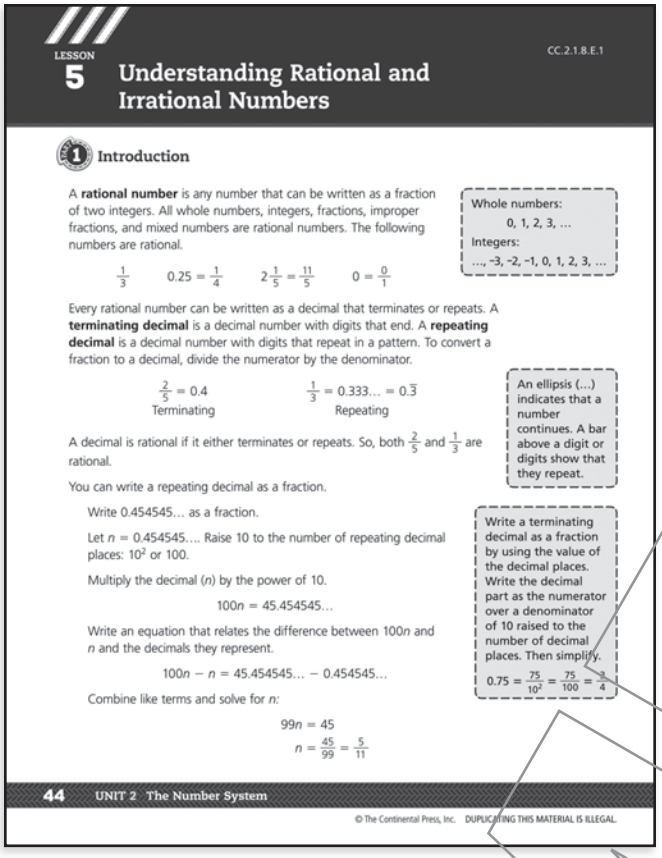

LESSON 5

Understanding Rational and Irrational Numbers

CC.2.1.8.E.1

1 Introduction

A **rational number** is any number that can be written as a fraction of two integers. All whole numbers, integers, fractions, improper fractions, and mixed numbers are rational numbers. The following numbers are rational.

$\frac{1}{3}$ $0.25 = \frac{1}{4}$ $2\frac{1}{5} = \frac{11}{5}$ $0 = \frac{0}{1}$

Whole numbers: 0, 1, 2, 3, ...
Integers: ..., −3, −2, −1, 0, 1, 2, 3, ...

Every rational number can be written as a decimal that terminates or repeats. A **terminating decimal** is a decimal number with digits that end. A **repeating decimal** is a decimal number with digits that repeat in a pattern. To convert a fraction to a decimal, divide the numerator by the denominator.

$\frac{2}{5} = 0.4$ Terminating $\frac{1}{3} = 0.333\ldots = 0.\overline{3}$ Repeating

An ellipsis (...) indicates that a number continues. A bar above a digit or digits show that they repeat.

A decimal is rational if it either terminates or repeats. So, both $\frac{2}{5}$ and $\frac{1}{3}$ are rational.

You can write a repeating decimal as a fraction.

Write 0.454545... as a fraction.

Let $n = 0.454545\ldots$. Raise 10 to the number of repeating decimal places: 10^2 or 100.

Multiply the decimal (n) by the power of 10.

$100n = 45.454545\ldots$

Write an equation that relates the difference between $100n$ and n and the decimals they represent.

$100n - n = 45.454545\ldots - 0.454545\ldots$

Combine like terms and solve for n:

$99n = 45$

$n = \frac{45}{99} = \frac{5}{11}$

Write a terminating decimal as a fraction by using the value of the decimal places. Write the decimal part as the numerator over a denominator of 10 raised to the number of decimal places. Then simplify.

$0.75 = \frac{75}{10^2} = \frac{75}{100} = \frac{3}{4}$

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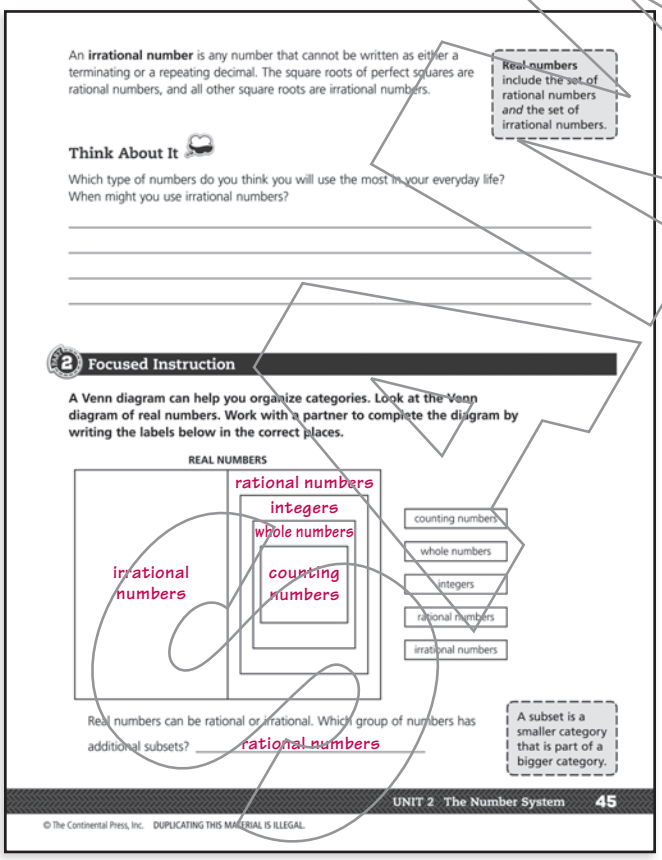

An **irrational number** is any number that cannot be written as either a terminating or a repeating decimal. The square roots of perfect squares are rational numbers, and all other square roots are irrational numbers.

Real numbers include the set of rational numbers *and* the set of irrational numbers.

Think About It

Which type of numbers do you think you will use the most in your everyday life? When might you use irrational numbers?

2 Focused Instruction

A Venn diagram can help you organize categories. Look at the Venn diagram of real numbers. Work with a partner to complete the diagram by writing the labels below in the correct places.

REAL NUMBERS

rational numbers
integers
whole numbers
counting numbers
irrational numbers

counting numbers
whole numbers
integers
rational numbers
irrational numbers

Real numbers can be rational or irrational. Which group of numbers has additional subsets? rational numbers

A subset is a smaller category that is part of a bigger category.

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Objective

To differentiate between rational and irrational numbers and terminating and repeating decimals, and to write repeating decimals as equivalent fractions

PART 1 Introduction

Review the difference between rational and irrational numbers, as well as the subcategories of rational numbers: counting numbers, whole numbers, and integers. Discuss terminating and repeating decimals. Then work through the sample item to convert a repeating decimal to a fraction.

Think About It

Students should recognize that they use counting numbers and whole numbers most in everyday life. They should recognize that irrational numbers will likely be used more in science and geometry.

PA Core Standard

CC.2.1.8.E.1 Distinguish between rational and irrational numbers using their properties.

Eligible Content

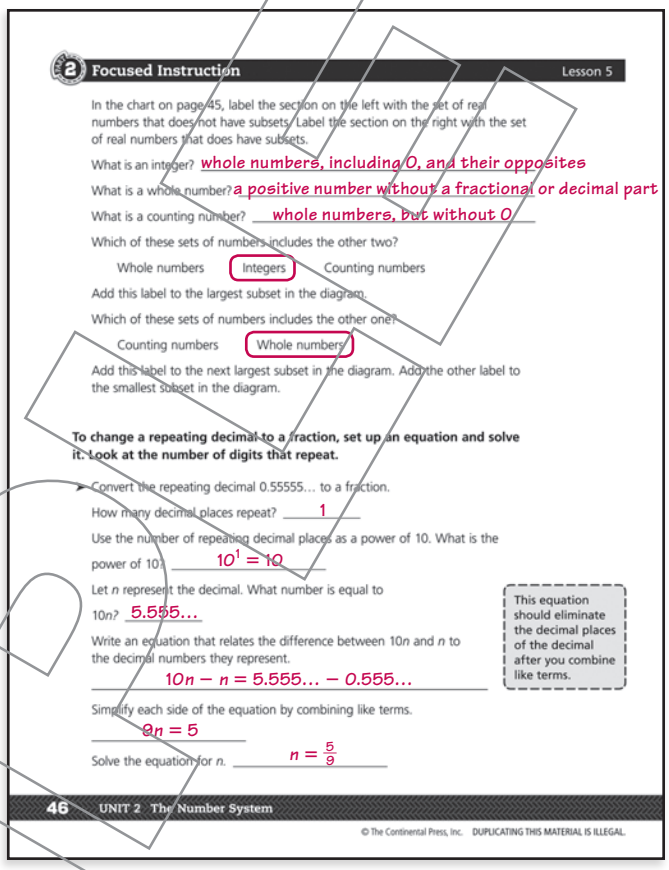
M08.A-N.1.1.1 Determine whether a number is rational or irrational. For rational numbers, show that the decimal expansion terminates or repeats (limit repeating decimals to thousandths).

M08.A-N.1.1.2 Convert a terminating or repeating decimal to a rational number (limit repeating decimals to thousandths).

Focused Instruction

First, students will use a Venn diagram to organize and understand the categories of real numbers. Next, students will change a repeating decimal into an equivalent fraction. Students must understand the difference between changing a terminating decimal into a fraction and changing a repeating decimal. They will work through a series of questions and use their knowledge of decimal place value.

Conclude the Focused Instruction section by having students solving two problems involving converting decimals to fractions.



Focused Instruction Lesson 5

In the chart on page 45, label the section on the left with the set of real numbers that does not have subsets. Label the section on the right with the set of real numbers that does have subsets.

What is an integer? whole numbers, including 0, and their opposites

What is a whole number? a positive number without a fractional or decimal part

What is a counting number? whole numbers, but without 0

Which of these sets of numbers includes the other two?

Whole numbers [Integers] Counting numbers

Add this label to the largest subset in the diagram.

Which of these sets of numbers includes the other one?

Counting numbers [Whole numbers]

Add this label to the next largest subset in the diagram. Add the other label to the smallest subset in the diagram.

To change a repeating decimal to a fraction, set up an equation and solve it. Look at the number of digits that repeat.

➤ Convert the repeating decimal 0.55555... to a fraction.

How many decimal places repeat? 1

Use the number of repeating decimal places as a power of 10. What is the power of 10? $10^1 = 10$

Let n represent the decimal. What number is equal to $10n$? 5.555...

Write an equation that relates the difference between $10n$ and n to the decimal numbers they represent. $10n - n = 5.555... - 0.555...$

This equation should eliminate the decimal places of the decimal after you combine like terms.

Simplify each side of the equation by combining like terms. $9n = 5$

Solve the equation for n. $n = \frac{5}{9}$

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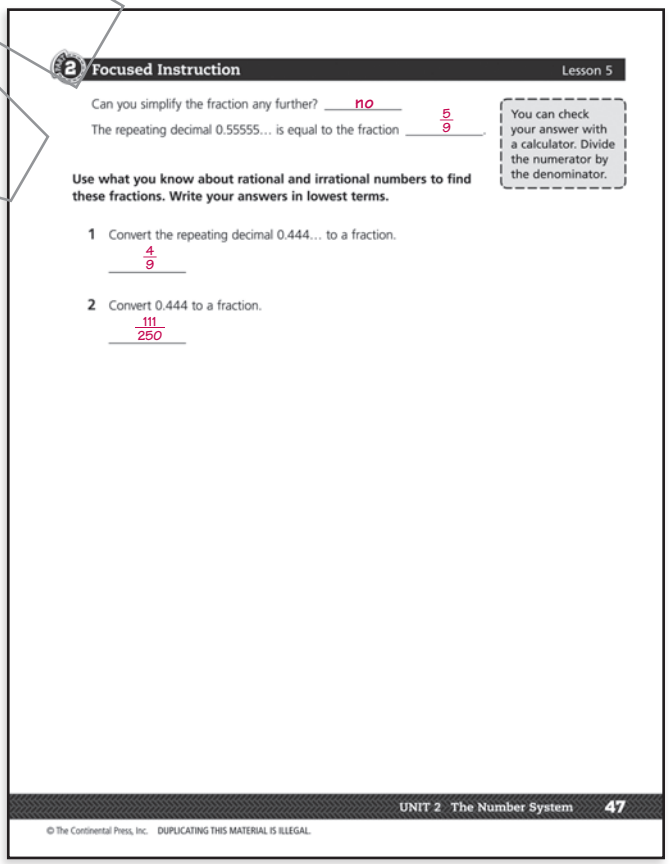

Focused Instruction Lesson 5

Can you simplify the fraction any further? no

The repeating decimal 0.55555... is equal to the fraction $\frac{5}{9}$.

You can check your answer with a calculator. Divide the numerator by the denominator.

Use what you know about rational and irrational numbers to find these fractions. Write your answers in lowest terms.

1 Convert the repeating decimal 0.444... to a fraction. $\frac{4}{9}$

2 Convert 0.444 to a fraction. $\frac{111}{250}$

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Vocabulary

irrational number: a real number that cannot be written in fraction form; a decimal that is non-terminating and non-repeating

rational number: any number that can be written as a fraction, including whole numbers, integers, fractions, and some decimals

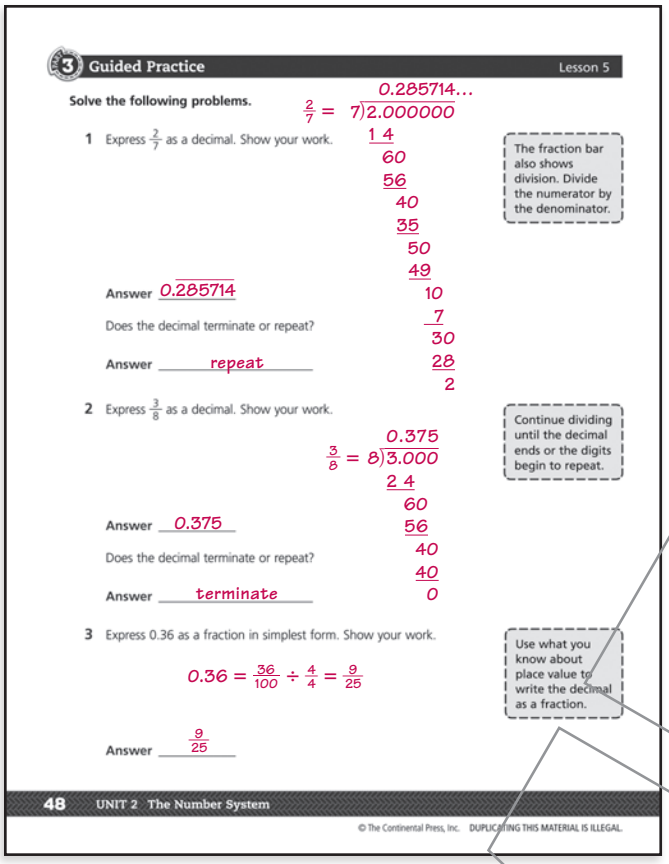
real numbers: the set of rational and irrational numbers

repeating decimal: a decimal in which digits repeat in a pattern

terminating decimal: a decimal whose digits end

Connections to Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Attend to precision.
- Look for and make use of structure.



3 Guided Practice — Lesson 5

Solve the following problems.

1 Express $\frac{2}{7}$ as a decimal. Show your work.

$\frac{2}{7} = 7\overline{)2.000000}$ = 0.285714...

14
60
56
40
35
50
49
10
7
30
28
2

The fraction bar also shows division. Divide the numerator by the denominator.

Answer 0.285714

Does the decimal terminate or repeat?

Answer repeat

2 Express $\frac{3}{8}$ as a decimal. Show your work.

$\frac{3}{8} = 8\overline{)3.000}$ = 0.375

24
60
56
40
40
0

Continue dividing until the decimal ends or the digits begin to repeat.

Answer 0.375

Does the decimal terminate or repeat?

Answer terminate

3 Express 0.36 as a fraction in simplest form. Show your work.

$0.36 = \frac{36}{100} \div \frac{4}{4} = \frac{9}{25}$

Use what you know about place value to write the decimal as a fraction.

Answer $\frac{9}{25}$

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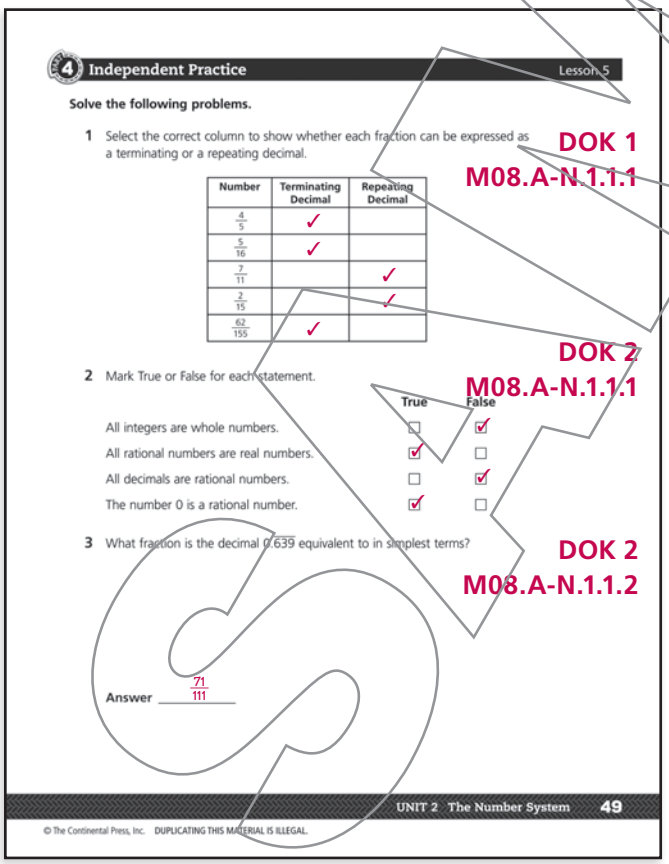

4 Independent Practice — Lesson 5

Solve the following problems.

1 Select the correct column to show whether each fraction can be expressed as a terminating or a repeating decimal.

DOK 1
M08.A-N.1.1.1

Number	Terminating Decimal	Repeating Decimal
$\frac{4}{5}$	✓	
$\frac{5}{16}$	✓	
$\frac{7}{11}$		✓
$\frac{2}{15}$		✓
$\frac{62}{155}$	✓	

2 Mark True or False for each statement.

DOK 2
M08.A-N.1.1.1

	True	False
All integers are whole numbers.	☐	☑
All rational numbers are real numbers.	☑	☐
All decimals are rational numbers.	☐	☑
The number 0 is a rational number.	☑	☐

3 What fraction is the decimal $0.\overline{639}$ equivalent to in simplest terms?

DOK 2
M08.A-N.1.1.2

Answer $\frac{71}{111}$

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PART 3 Guided Practice

Students should complete the Guided Practice section on their own. Offer assistance as needed, pointing out the reminder and hint boxes along the right side of the page.

PART 4 Independent Practice Answer Rationales

1 The numbers $\frac{4}{5}$, $\frac{5}{16}$, and $\frac{62}{155}$ represent terminating decimals because their decimals end: $\frac{4}{5} = 0.8$, $\frac{5}{16} = 0.3125$, and $\frac{62}{155} = 0.4$. The fractions $\frac{7}{11}$ and $\frac{2}{15}$ are equivalent to repeating decimals because one or more digits continue forever: $\frac{7}{11} = 0.636363\ldots$ and $\frac{2}{15} = 0.133333\ldots$.

2 Integers include negative numbers, so not all integers are whole numbers. The first statement is false. The second statement is true. Real numbers are composed of all rational numbers and all irrational numbers. The third statement is false because non-terminating, non-repeating decimals are not rational. The fourth statement is true. The number 0 can be written as a fraction $\frac{0}{0}$, so it is a rational number.

3 Let $n = 0.\overline{639}$. There are three repeating decimals places, so raise 10 to a power of 3: 10^3, or 1,000. Write the equation relating the difference between $1{,}000n$ and n and the decimals they represent: $1{,}000n - n = 639.639639\ldots - 0.639639$. Combine like terms and solve for n: $999n = 639$; $n = \frac{639}{999}$. Simplify by dividing the numerator and the denominator by 9: $\frac{639}{999} \div \frac{9}{9} = \frac{71}{111}$.

Extension Activity

Make a number chart based off the Venn diagram in the Focused Instruction section. Work as a class to come up with examples of each type of number and post the chart in the classroom for future reference.

4 Choices C and D are correct; 0.213568… and 0.030405… are both irrational because they do not end and they do not repeat. The remaining choices are rational because they represent either terminating or repeating decimals. Choice A is a repeating decimal. Choice B is the ratio of two integers. Choice E is a terminating decimal. Choice F is the ratio of two integers.

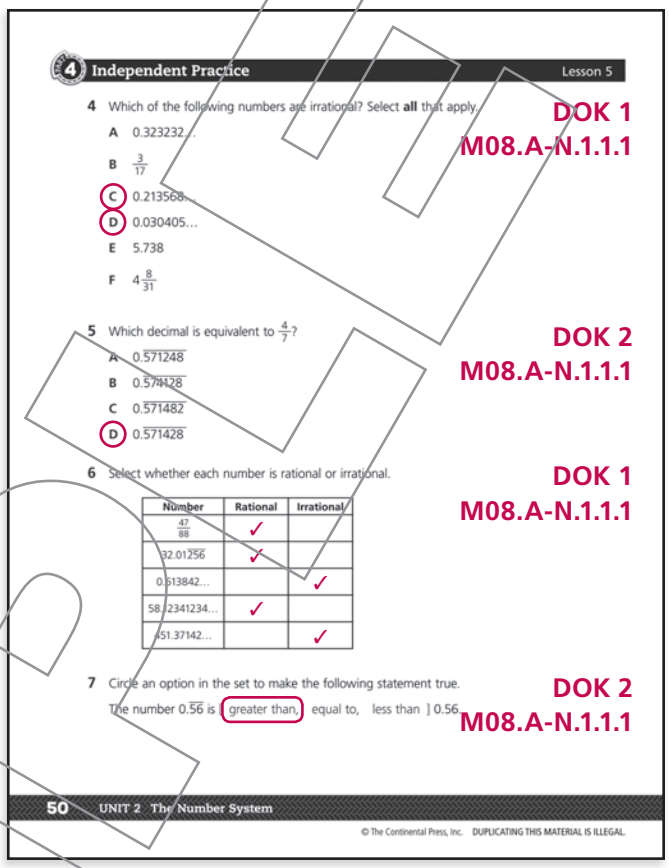
5 To change $\frac{4}{7}$ to a decimal, divide the numerator by the denominator. Continue dividing until the digits end or begin to repeat:

$$
\begin{array}{r}
0.571428\ldots \\
7\overline{)\,4.000000} \\
\underline{35} \\
50 \\
\underline{49} \\
10 \\
\underline{7} \\
30 \\
\underline{28} \\
20 \\
\underline{14} \\
60 \\
\underline{56} \\
4
\end{array}
$$

So $\frac{4}{7}$ is equivalent to the decimal $0.\overline{571428}$, choice D.

6 The first, second, and fourth numbers are rational. The first number, $\frac{47}{88}$, is the ratio of two integers. The second number, $32.01\overline{256}$, is a repeating decimal. The fourth number, 58.12341234…, is a repeating decimal. The third and fifth numbers are irrational because the decimals do not end and do not repeat.

7 The student should write out more decimal places to see that the repeating decimal is larger: $0.565656\ldots > 0.560000\ldots$.



4 Independent Practice — Lesson 5

4 Which of the following numbers are irrational? Select **all** that apply. — DOK 1, M08.A-N.1.1.1

A 0.323232…
B $\frac{3}{17}$
C 0.213568…
D 0.030405…
E 5.738
F $4\frac{8}{31}$

5 Which decimal is equivalent to $\frac{4}{7}$? — DOK 2, M08.A-N.1.1.1

A $0.\overline{571248}$
B $0.\overline{574128}$
C $0.\overline{571482}$
D $0.\overline{571428}$

6 Select whether each number is rational or irrational. — DOK 1, M08.A-N.1.1.1

Number	Rational	Irrational
$\frac{47}{88}$	✓	
$32.01\overline{256}$	✓	
0.513842…		✓
58.12341234…	✓	
51.37142…		✓

7 Circle an option in the set to make the following statement true. — DOK 2, M08.A-N.1.1.1

The number $0.\overline{56}$ is (greater than) equal to, less than] 0.56.

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