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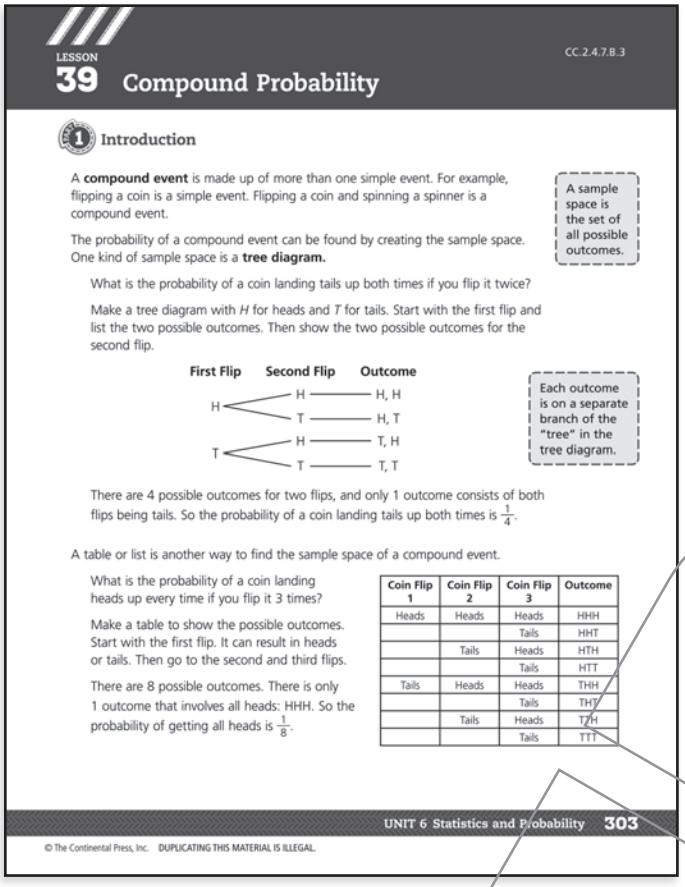

LESSON 39 Compound Probability

CC.2.4.7.B.3

1 Introduction

A **compound event** is made up of more than one simple event. For example, flipping a coin is a simple event. Flipping a coin and spinning a spinner is a compound event.

A sample space is the set of all possible outcomes.

The probability of a compound event can be found by creating the sample space. One kind of sample space is a **tree diagram.**

What is the probability of a coin landing tails up both times if you flip it twice?

Make a tree diagram with *H* for heads and *T* for tails. Start with the first flip and list the two possible outcomes. Then show the two possible outcomes for the second flip.

First Flip	Second Flip	Outcome
H	H	H, H
	T	H, T
T	H	T, H
	T	T, T

Each outcome is on a separate branch of the "tree" in the tree diagram.

There are 4 possible outcomes for two flips, and only 1 outcome consists of both flips being tails. So the probability of a coin landing tails up both times is $\frac{1}{4}$.

A table or list is another way to find the sample space of a compound event.

What is the probability of a coin landing heads up every time if you flip it 3 times?

Make a table to show the possible outcomes. Start with the first flip. It can result in heads or tails. Then go to the second and third flips.

There are 8 possible outcomes. There is only 1 outcome that involves all heads: HHH. So the probability of getting all heads is $\frac{1}{8}$.

Coin Flip 1	Coin Flip 2	Coin Flip 3	Outcome
Heads	Heads	Heads	HHH
		Tails	HHT
	Tails	Heads	HTH
		Tails	HTT
Tails	Heads	Heads	THH
		Tails	THT
	Tails	Heads	TTH
		Tails	TTT

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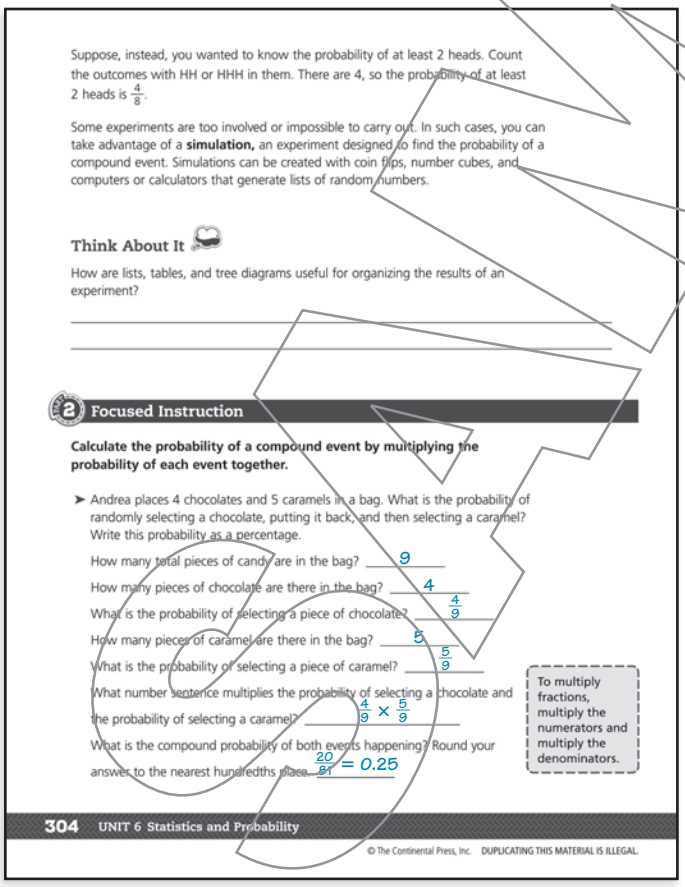

Suppose, instead, you wanted to know the probability of at least 2 heads. Count the outcomes with HH or HHH in them. There are 4, so the probability of at least 2 heads is $\frac{4}{8}$.

Some experiments are too involved or impossible to carry out. In such cases, you can take advantage of a **simulation,** an experiment designed to find the probability of a compound event. Simulations can be created with coin flips, number cubes, and computers or calculators that generate lists of random numbers.

Think About It

How are lists, tables, and tree diagrams useful for organizing the results of an experiment?

2 Focused Instruction

Calculate the probability of a compound event by multiplying the probability of each event together.

➤ Andrea places 4 chocolates and 5 caramels in a bag. What is the probability of randomly selecting a chocolate, putting it back, and then selecting a caramel? Write this probability as a percentage.

How many total pieces of candy are in the bag? 9

How many pieces of chocolate are there in the bag? 4

What is the probability of selecting a piece of chocolate? $\frac{4}{9}$

How many pieces of caramel are there in the bag? 5

What is the probability of selecting a piece of caramel? $\frac{5}{9}$

What number sentence multiplies the probability of selecting a chocolate and the probability of selecting a caramel? $\frac{4}{9} \times \frac{5}{9}$

To multiply fractions, multiply the numerators and multiply the denominators.

What is the compound probability of both events happening? Round your answer to the nearest hundredths place. $\frac{20}{81} = 0.25$

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Objective

To review finding compound probability and using a sample space and simulation

PART 1 Introduction

Discuss compound probability events as those in which two or more simple events occur. A sample space can represent the outcomes of a compound event; it combines all the possible outcomes of the first event with all the possible outcomes of the second event. Work through the examples on the page of sample spaces in the forms of tree diagrams and tables.

Then discuss the use of simulations, an experiment that replicates the random outcomes of a compound event. If available, use a random number generator on a computer or calculator to demonstrate.

Think About It

Students should recognize that when the outcomes are systematically recorded, the organizers help to account for all possible combinations of the first event and the subsequent events.

PA Core Standard

CC.2.4.7.B.3 Investigate chance processes and develop, use, and evaluate probability models.

Eligible Content

M07.D-S.3.2.3 Find probabilities of independent compound events using organized lists, tables, tree diagram, and simulation.

Vocabulary

compound event: an event consisting of two or more events occurring together

simulation: an experiment used to find the probability of a compound event

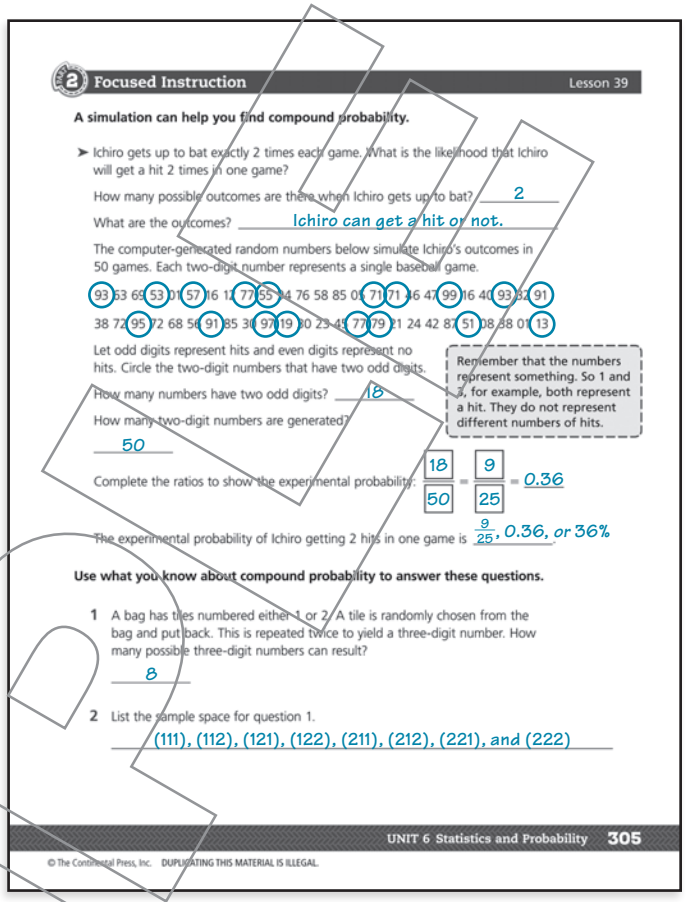
tree diagram: a visual representation of a sample space using branches

Part 2 Focused Instruction

First, students find the compound probability of two events mathematically by multiplying the probability of one event by the probability of the second event.

Next, students use a simulation to find the experimental probability of a compound event. They are given a set of 50 two-digit numbers to represent hits/no hits in 50 baseball games. Students interpret the simulation and find the probability.

Conclude the Focused Instruction section by having students find the number of possible outcomes for a compound event and list the sample space for it.



Part 2 Focused Instruction — Lesson 39

A simulation can help you find compound probability.

➤ Ichiro gets up to bat exactly 2 times each game. What is the likelihood that Ichiro will get a hit 2 times in one game?

How many possible outcomes are there when Ichiro gets up to bat? 2

What are the outcomes? Ichiro can get a hit or not.

The computer-generated random numbers below simulate Ichiro's outcomes in 50 games. Each two-digit number represents a single baseball game.

93 63 69 53 01 57 16 11 77 55 44 76 58 85 05 71 71 46 47 99 16 40 93 52 91
38 72 95 72 68 56 91 85 30 97 19 80 23 45 77 79 21 24 42 87 51 08 88 01 13

Let odd digits represent hits and even digits represent no hits. Circle the two-digit numbers that have two odd digits.

How many numbers have two odd digits? 18

How many two-digit numbers are generated? 50

Remember that the numbers represent something. So 1 and 3, for example, both represent a hit. They do not represent different numbers of hits.

Complete the ratios to show the experimental probability: $\frac{18}{50} = \frac{9}{25} = 0.36$

The experimental probability of Ichiro getting 2 hits in one game is $\frac{9}{25}$, 0.36, or 36%

Use what you know about compound probability to answer these questions.

1 A bag has tiles numbered either 1 or 2. A tile is randomly chosen from the bag and put back. This is repeated twice to yield a three-digit number. How many possible three-digit numbers can result?

8

2 List the sample space for question 1.

(111), (112), (121), (122), (211), (212), (221), and (222)

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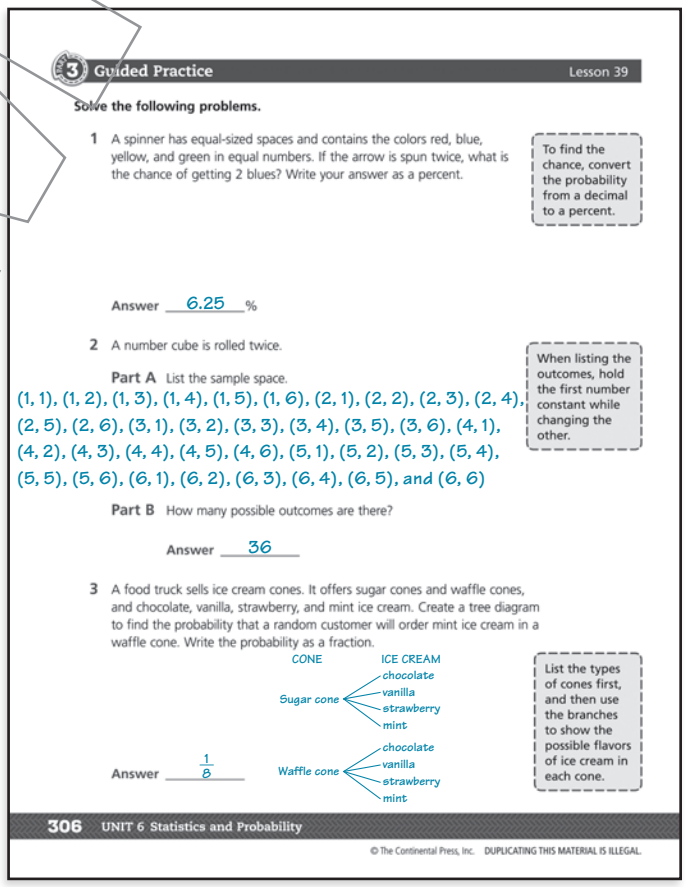
Part 3 Guided Practice

Students should complete the Guided Practice section on their own. Offer assistance as needed, pointing out the reminder and hint boxes along the right side of the page.



Part 3 Guided Practice — Lesson 39

Solve the following problems.

1 A spinner has equal-sized spaces and contains the colors red, blue, yellow, and green in equal numbers. If the arrow is spun twice, what is the chance of getting 2 blues? Write your answer as a percent.

To find the chance, convert the probability from a decimal to a percent.

Answer 6.25 %

2 A number cube is rolled twice.

Part A List the sample space.

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), and (6, 6)

When listing the outcomes, hold the first number constant while changing the other.

Part B How many possible outcomes are there?

Answer 36

3 A food truck sells ice cream cones. It offers sugar cones and waffle cones, and chocolate, vanilla, strawberry, and mint ice cream. Create a tree diagram to find the probability that a random customer will order mint ice cream in a waffle cone. Write the probability as a fraction.

CONE — ICE CREAM

Sugar cone: chocolate, vanilla, strawberry, mint

Waffle cone: chocolate, vanilla, strawberry, mint

List the types of cones first, and then use the branches to show the possible flavors of ice cream in each cone.

Answer $\frac{1}{8}$

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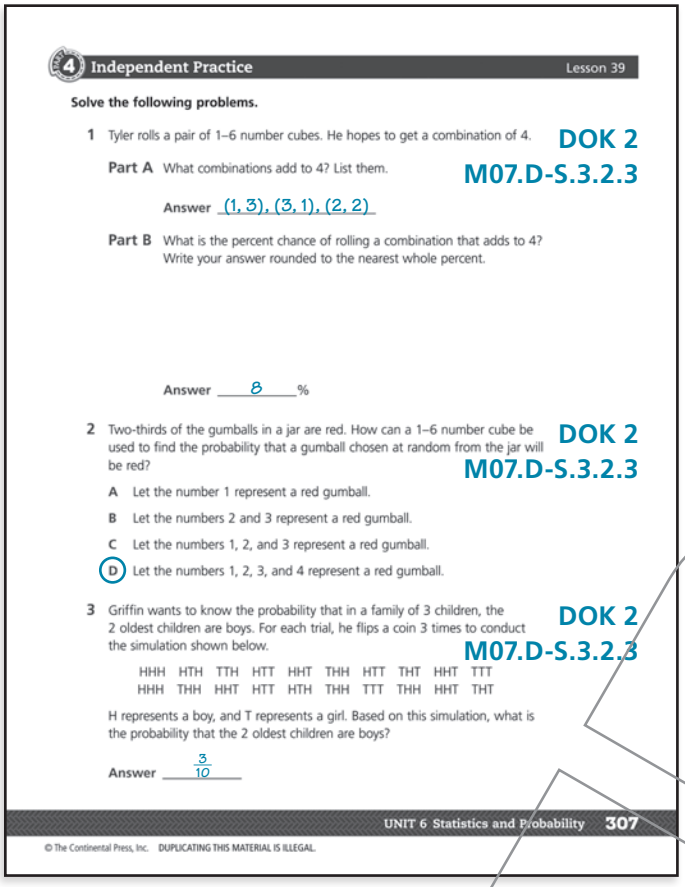
Connections to Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Model with mathematics.
- Attend to precision.
- Look for and make use of structure.



4 Independent Practice Lesson 39

Solve the following problems.

1 Tyler rolls a pair of 1–6 number cubes. He hopes to get a combination of 4. DOK 2 M07.D-S.3.2.3

Part A What combinations add to 4? List them.

Answer (1, 3), (3, 1), (2, 2)

Part B What is the percent chance of rolling a combination that adds to 4? Write your answer rounded to the nearest whole percent.

Answer 8 %

2 Two-thirds of the gumballs in a jar are red. How can a 1–6 number cube be used to find the probability that a gumball chosen at random from the jar will be red? DOK 2 M07.D-S.3.2.3

A Let the number 1 represent a red gumball.

B Let the numbers 2 and 3 represent a red gumball.

C Let the numbers 1, 2, and 3 represent a red gumball.

(D) Let the numbers 1, 2, 3, and 4 represent a red gumball.

3 Griffin wants to know the probability that in a family of 3 children, the 2 oldest children are boys. For each trial, he flips a coin 3 times to conduct the simulation shown below. DOK 2 M07.D-S.3.2.3

HHH HTH TTH HTT HHT THH HTT THT HHT TTT
HHH THH HHT HTT HTH THH TTT THH HHT THT

H represents a boy, and T represents a girl. Based on this simulation, what is the probability that the 2 oldest children are boys?

Answer $\frac{3}{10}$

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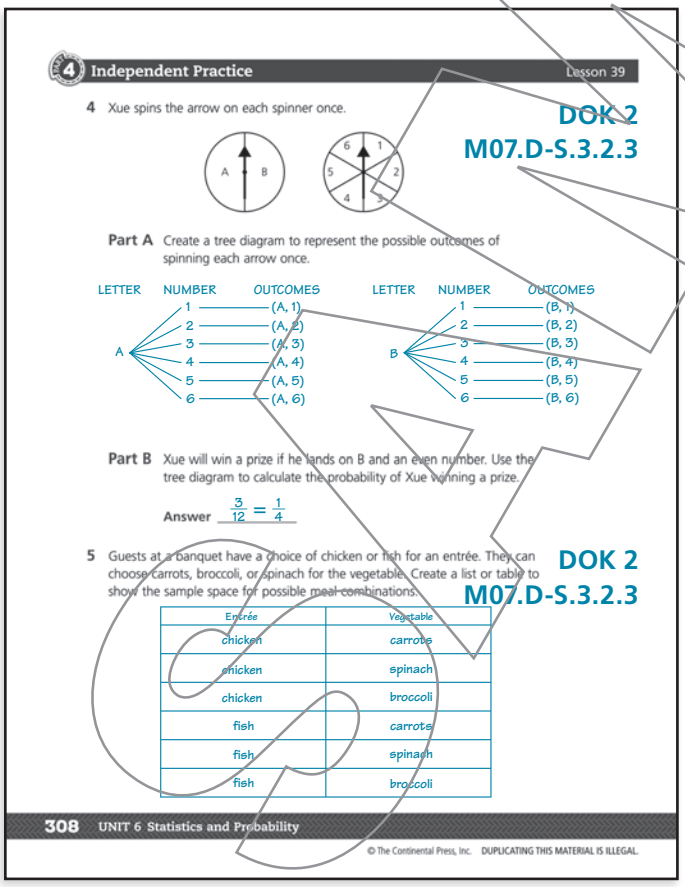

4 Independent Practice Lesson 39

4 Xue spins the arrow on each spinner once. DOK 2 M07.D-S.3.2.3

Part A Create a tree diagram to represent the possible outcomes of spinning each arrow once.

LETTER	NUMBER	OUTCOMES
A	1	(A, 1)
	2	(A, 2)
	3	(A, 3)
	4	(A, 4)
	5	(A, 5)
	6	(A, 6)
B	1	(B, 1)
	2	(B, 2)
	3	(B, 3)
	4	(B, 4)
	5	(B, 5)
	6	(B, 6)

Part B Xue will win a prize if he lands on B and an even number. Use the tree diagram to calculate the probability of Xue winning a prize.

Answer $\frac{3}{12} = \frac{1}{4}$

5 Guests at a banquet have a choice of chicken or fish for an entrée. They can choose carrots, broccoli, or spinach for the vegetable. Create a list or table to show the sample space for possible meal combinations. DOK 2 M07.D-S.3.2.3

Entrée	Vegetable
chicken	carrots
chicken	spinach
chicken	broccoli
fish	carrots
fish	spinach
fish	broccoli

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PART 4 Independent Practice Answer Rationales

1 PART A There are three possible ways to make 4 with a pair of 1–6 number cubes: (1, 3), (3, 1), and (2, 2).

PART B Each number cube can land 6 ways, so there are $6 \times 6 = 36$ possible outcomes in all. Only 3 outcomes result in a sum of 4, which is a probability of 3 out of 36 or $\frac{3}{36} = \frac{1}{12}$. Change the fraction to a decimal, $0.08\overline{3}$, and finally change the decimal to the nearest whole percent, 8%.

2 A six-sided number cube is a good simulator of gumballs in a jar because two-thirds of the gumballs can be easily represented by $\frac{2}{3}$ of the numbers on the cube, that is, by four of the numbers, such as 1, 2, 3, and 4. Choice D is correct. Choice A is incorrect because it would show $\frac{1}{6}$ of the gumballs. Choice B is incorrect because it would show $\frac{2}{6}$ or $\frac{1}{3}$ of the gumballs. Choice C is incorrect because it would show $\frac{3}{6}$ or $\frac{1}{2}$ of the gumballs.

3 Count the number of outcomes that have H as the first two results. There is a total of 6 outcomes with either HHH or HHT. The total number of trials is 20. So, the probability is $\frac{6}{20} = \frac{3}{10}$.

4 PART A Show the two outcomes (A and B) of the first spinner first. From each outcome, show six branches for the outcomes (1, 2, 3, 4, 5, and 6) of the second spinner.

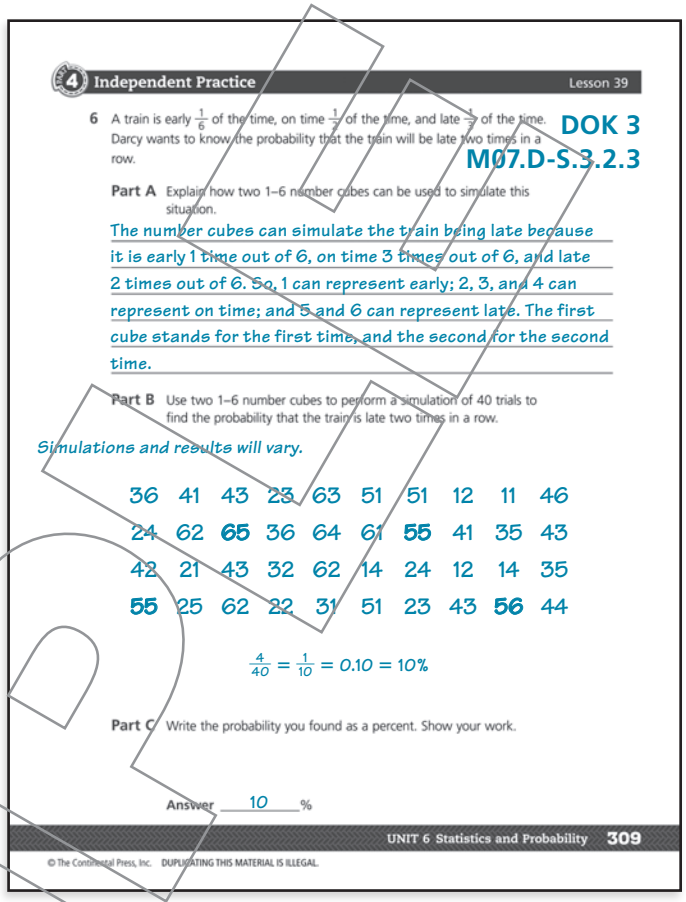
PART B There are 12 possible outcomes on the tree diagram. There are 3 outcomes that involve B and an even number: (B, 2), (B, 4), and (B, 6). The probability is the number of possible outcomes divided by the number of total outcomes: $\frac{3}{12} = \frac{1}{4}$.

5 In a table or list, write each meat entrée paired with one of the three vegetable choices. There should be a total of 6 pairings, organized in the same way.

6 PART A The fraction $\frac{1}{2}$ is equivalent to $\frac{3}{6}$ and $\frac{1}{3}$ is equivalent to $\frac{2}{6}$. Therefore, a 1–6 number cube can represent these fractions in a simulation. The train is early $\frac{1}{6}$ of the time; this outcome can be shown with one number on the cube. The train is on time $\frac{3}{6}$ of the time, which can be shown using three numbers of the cube. A late train can be represented with two numbers on the cube. For example, 1 = early; 2, 3, and 4 = on time; and 5 and 6 = late.

PART B Toss a pair of number cubes 40 times to simulate two consecutive arrivals. Late two times in a row would be represented by the combinations (5, 5), (5, 6), (6, 5), and (6, 6). Count the number of occurrences in 40 trials and calculate the experimental probability.

PART C To express the probability as a percent, divide the numerator of the fraction by the denominator for a decimal and omit the decimal point.



4 Independent Practice Lesson 39

6 A train is early $\frac{1}{6}$ of the time, on time $\frac{1}{2}$ of the time, and late $\frac{1}{3}$ of the time. Darcy wants to know the probability that the train will be late two times in a row. DOK 3 M07.D-S.3.2.3

Part A Explain how two 1–6 number cubes can be used to simulate this situation.

The number cubes can simulate the train being late because it is early 1 time out of 6, on time 3 times out of 6, and late 2 times out of 6. So, 1 can represent early; 2, 3, and 4 can represent on time; and 5 and 6 can represent late. The first cube stands for the first time, and the second for the second time.

Part B Use two 1–6 number cubes to perform a simulation of 40 trials to find the probability that the train is late two times in a row.

Simulations and results will vary.

36	41	43	23	63	51	51	12	11	46
24	62	**65**	36	64	61	**55**	41	35	43
42	21	43	32	62	14	24	12	14	35
55	25	62	22	31	51	23	43	**56**	44

$\frac{4}{40} = \frac{1}{10} = 0.10 = 10\%$

Part C Write the probability you found as a percent. Show your work.

Answer 10 %

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Extension Activity

Have small groups of students design and conduct a simulation to find the probability of a compound event, such as the following situation. At a steak restaurant, $\frac{1}{6}$ of the diners order their steak cooked rare, $\frac{1}{2}$ order medium, and $\frac{1}{3}$ order well done. What is the probability that a table of 3 diners will all order their steak the same way (RRR, MMM, or WWW)?

