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LESSON
8 Polynomial Expressions

A **polynomial** is an algebraic expression of one or more **terms,** which are parts of an expression separated by the operations of addition or subtraction. A **monomial** is a polynomial with only one term, like 8, 8*y,* or $8y^2$. A **binomial** is a polynomial with two terms, like $6x - 7$. A **trinomial** has three terms, like $x^2 + 3 - 10$. Terms of a polynomial can be variables that are raised to whole-number powers, such as x^2 or $6y^3z$. The exponents, however, must be non-negative for the expression to be a polynomial. For example, $2x^{-2}$ is not a polynomial.

> A polynomial has no variable in the denominator.
> $\frac{5}{x}$ is not a polynomial.
> A variable written without an exponent has a power of 1.
> $x = x^1$

Adding and Subtracting Polynomials

To add polynomials, simply combine like terms. See how it works in this example:

$(x^2 - 9x + 3) + (2x^2 + x - 4)$
$x^2 + 2x^2 - 9x + x + 3 - 4$
$3x^2 - 8x - 1$

To subtract polynomials, change the sign of each term in the second polynomial and then add. For example,

$(2x^3 + 4x^2 - 6x) - (4x^3 - 3x^2 + 5x)$

is written as

$(2x^3 + 4x^2 - 6x) + (-4x^3 + 3x^2 - 5x)$

which equals

$-2x^3 + 7x^2 - 11x.$

> Subtracting is adding the opposite.
> $a - b = a + (-b)$

Try this sample question. SAMPLE 1

What is $(4x^2 - 6) - (x^2 + 2)$ in simplest form?

A $3x^2 - 8$ **B** $3x^2 - 4$ **C** $5x^2 - 8$ **D** $5x^2 - 4$

Changing the sign of each term in the second polynomial gives the addition problem $(4x^2 - 6) + (-x^2 - 2)$, which simplifies to $3x^2 - 8$. Choice A is correct.

Try this sample question. SAMPLE 2

Write the simplified form of this expression.

$$(6x^2 + 4y^2) + (5x^2 - 3y^2)$$

To add these binomials, simply combine like terms: $6x^2 + 5x^2 = 11x^2$ and $4y^2 - 3y^2 = y^2$. So the simplified form is $11x^2 + y^2$.

Multiplying Polynomials

To multiply polynomial expressions, multiply each term in the first polynomial by each term in the second polynomial. Then combine like terms, if needed, to simplify the resulting polynomial expression. For example:

$$\begin{aligned}(4x + 3)(x - 2) &= 4x(x - 2) + 3(x - 2)\\ &= 4x^2 - 8x + 3x - 6\\ &= 4x^2 - 5x - 6\end{aligned}$$

When multiplying variable terms with like bases, add the exponents.

$$x^5 \cdot x^2 = x^{5+2} = x^7$$

Try this sample question. SAMPLE 3

The length of a rectangle is $5x + 1$ units long. The width of the rectangle is $2x + 5$ units long. What is the area, in square units, of the rectangle?

A $7x^2 + 6$

B $10x^2 + 5$

C $10x^2 + 7x + 5$

D $10x^2 + 27x + 5$

The area of a rectangle is length × width. So, the area of this rectangle is $(5x + 1)(2x + 5)$. To find the area, multiply each term in $5x + 1$ by each term in $2x + 5$:

$$\begin{aligned}(5x + 1)(2x + 5) &= 5x(2x + 5) + 1(2x + 5)\\ &= 10x^2 + 25x + 2x + 5\end{aligned}$$

Combine like terms. This gives $10x^2 + 27x + 5$. Choice D is correct.

Dividing Polynomials by Monomials

Sometimes you may be asked to divide a polynomial expression by a monomial expression. Each term of the polynomial expression must be divided by the monomial.

When dividing variable terms with like bases, subtract the exponents.
$x^5 \div x^2 = x^{5-2} = x^3$

In the example below, the monomial $4p$ divides each term of the polynomial, creating separate fractions. Then common terms are canceled within each fraction to simplify the expression.

$$\frac{16p^5 - 4p^3 - 12p^2}{4p} = \frac{16p^5}{4p} - \frac{4p^3}{4p} - \frac{12p^2}{4p}$$

$$= 4p^4 - p^2 - 3p$$

Try this sample question. SAMPLE 4

What is the simplified form of the expression below?

$$(9x^4 + 6x^3 - 15x^2) \div 3x^2$$

A $3x^2 + 2x - 5$

B $6x^2 + 3x - 12$

C $3x^2 + 6x^3 - 15x^2$

D $6x^2 + 6x^3 - 15x^2$

This item can be rewritten as the sum and difference of separate fractions:
$\frac{9x^4}{3x^2} + \frac{6x^3}{3x^2} - \frac{15x^2}{3x^2}$.
Simplifying each fraction gives $3x^2 + 2x - 5$. Choice A is correct.

Polynomials and Closure

Polynomials share a very important property with real numbers. When you add, subtract, or multiply any two real numbers, the result is always a real number. Likewise, when you add, subtract, or multiply any two polynomials, the result is always a polynomial. This property is called **closure,** and the set of all polynomials is closed under the operations of addition, subtraction, and multiplication.

For example, the sum of the polynomials $x^3 - 1$ and $2x + 5$ is the polynomial $x^3 + 2x + 4$, their difference is $x^3 - 2x - 6$, and their product is $2x^4 + 5x^3 - 2x - 5$. All three combinations yield a polynomial.

However, closure fails under division. Although sometimes dividing one polynomial by a second yields a third polynomial, this is not always the case.

For example, dividing the polynomial $x^2 + 3x + 2$ by the polynomial $x + 2$ yields the polynomial $x + 1$. However, dividing the polynomial $x^2 + 3x + 2$ by the polynomial x yields the expression $x + 3 + 2x^{-1}$. The term $2x^{-1}$ has a negative exponent, so this expression is not a polynomial.

Try this sample question. SAMPLE 5

For the polynomials $2x + 3$ and $6x^2 + x - 15$, verify that the sum, difference, and product are also polynomials.

The sum $(2x + 3) + (6x^2 + x - 15)$ simplifies to $6x^2 + 3x - 12$, the difference $(2x + 3) - (6x^2 + x - 15)$ simplifies to $^-6x^2 + x + 18$, and the product $(2x + 3)(6x^2 + x - 15)$ simplifies to $12x^3 + 20x^2 - 27x - 45$. All three resulting expressions have terms with only non-negative integer exponents, and are therefore also polynomials.

INDEPENDENT PRACTICE

Read and solve each problem.

1 What is the simplified form of this expression?

$$(2w^3 + 4w^2 + 5) + (3w^3 + w^2 + 4w)$$

A $5w^3 + 5w^2 + 9$

B $5w^3 + 5w^2 + 9w$

C $5w^3 + 5w^2 + 4w + 5$

D $5w^3 + 4w^2 + 5w + 4$

2 Write each expression in simplest form. Show your work.

A $5y^2(8y^3 - 2y^2 + 7y)$

C $(5x + 3)(x^2 - x - 4)$

B $3x(2x + y) - y(x + 2y - 1)$

D $(9y^2 - 4y + 3) - (5y^2 + 2y - 7)$

3 Write an expression equivalent to

$$-2s(4s - t + 3) - t(-3s + 5t) + 6(s - 2t - 3)$$

by entering the coefficients in each empty box.

☐ s^2 + ☐ t^2 + ☐ st + ☐ s + ☐ t + ☐

4 Simplify: $\frac{20x^8 + 16x^7 - 4x^6}{4x^6}$.

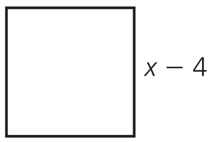
A $5x^2 + 4x - 1$

B $5x^2 + 16x - 4$

C $16x^2 + 12x - 1$

D $16x^2 + 16x - 4$

5 The diagram shows a square and the length of one side.



A Write an expression in simplest form that represents the perimeter of the square.

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B Write an expression in simplest form that represents the area of the square.

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6 Divide $15y^7 + 12y^6 - 9y^5$ by $3y^3$.

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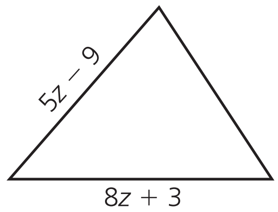
7 The perimeter of the triangle shown is $19z - 7$.



A Write an expression in simplest terms to represent the length of the remaining side. Show all steps in your work.

B The height of the triangle is represented by $2y + 3z$. Write an expression that models the area of the triangle. Show your work.