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Objective

To understand systems of equations and find a solution on a graph

Part 1 Introduction

Explain that a system of equations is two or more linear equations in a set and that the solution is the point where the equations meet. Students should become familiar with what a system of equations looks like and how to find the intersection point. Explain that some systems of equations are parallel lines, which means they do not have a solution. Others result in the same line, which means they have infinite solutions.

Think About It

Students should recognize that they would be able to tell certain things about the equations in a system of equations by writing all the equations in slope-intercept form. If the equations are identical, the lines are the same, so there are infinite solutions to the system of equations. If the equations have the same slope but different *y*-intercepts, the lines are parallel and there is no solution to the system of equations.

Georgia Standards of Excellence

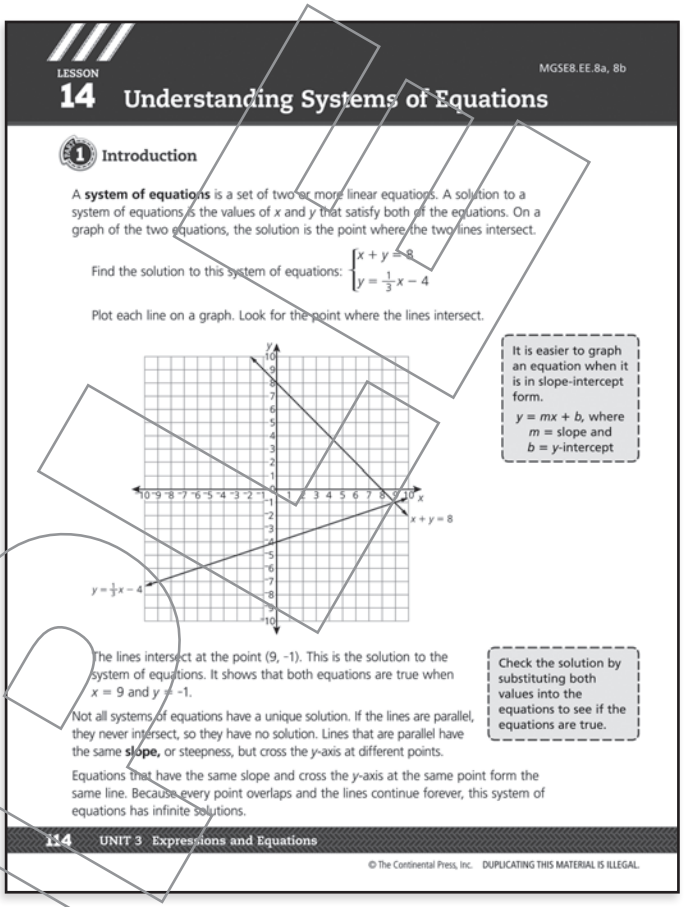
MGSE8.EE.8 Analyze and solve pairs of simultaneous linear equations (systems of linear equations).

- **8a.** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- **8b.** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

Vocabulary

slope: the steepness of a line that shows how the change in one variable relates to the change in the other variable

system of equations: a set of two or more equations



LESSON 14 Understanding Systems of Equations

MGSE8.EE.8a, 8b

1 Introduction

A **system of equations** is a set of two or more linear equations. A solution to a system of equations is the values of *x* and *y* that satisfy both of the equations. On a graph of the two equations, the solution is the point where the two lines intersect.

Find the solution to this system of equations: $\begin{cases} x + y = 8 \\ y = \frac{1}{3}x - 4 \end{cases}$

Plot each line on a graph. Look for the point where the lines intersect.

It is easier to graph an equation when it is in slope-intercept form.
$y = mx + b$, where m = slope and b = *y*-intercept

The lines intersect at the point (9, -1). This is the solution to the system of equations. It shows that both equations are true when $x = 9$ and $y = -1$.

Check the solution by substituting both values into the equations to see if the equations are true.

Not all systems of equations have a unique solution. If the lines are parallel, they never intersect, so they have no solution. Lines that are parallel have the same **slope,** or steepness, but cross the *y*-axis at different points.

Equations that have the same slope and cross the *y*-axis at the same point form the same line. Because every point overlaps and the lines continue forever, this system of equations has infinite solutions.

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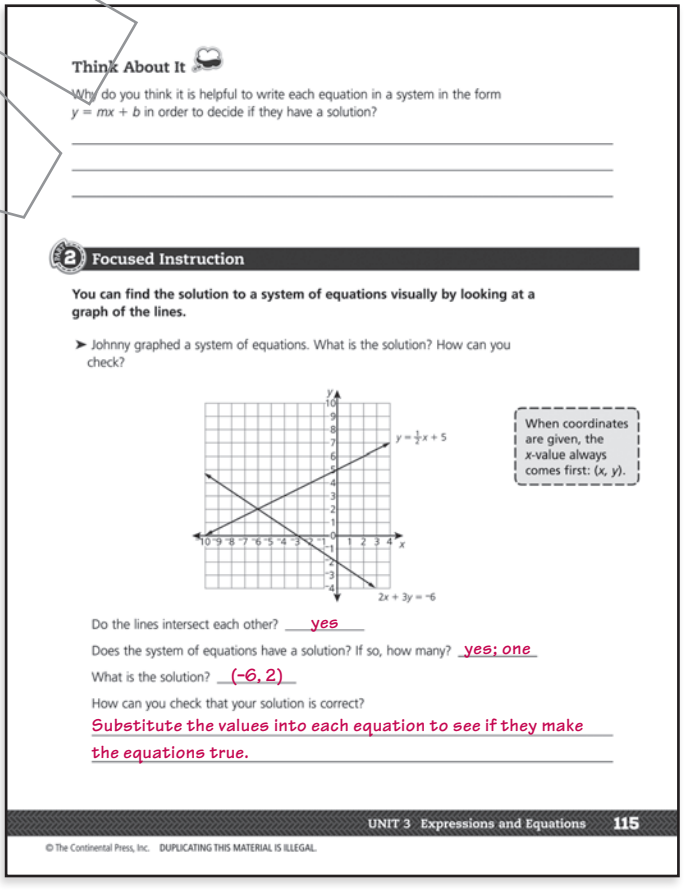

Think About It

Why do you think it is helpful to write each equation in a system in the form $y = mx + b$ in order to decide if they have a solution?

2 Focused Instruction

You can find the solution to a system of equations visually by looking at a graph of the lines.

➤ Johnny graphed a system of equations. What is the solution? How can you check?

When coordinates are given, the *x*-value always comes first: (*x*, *y*).

Do the lines intersect each other? yes

Does the system of equations have a solution? If so, how many? yes; one

What is the solution? (-6, 2)

How can you check that your solution is correct?

Substitute the values into each equation to see if they make the equations true.

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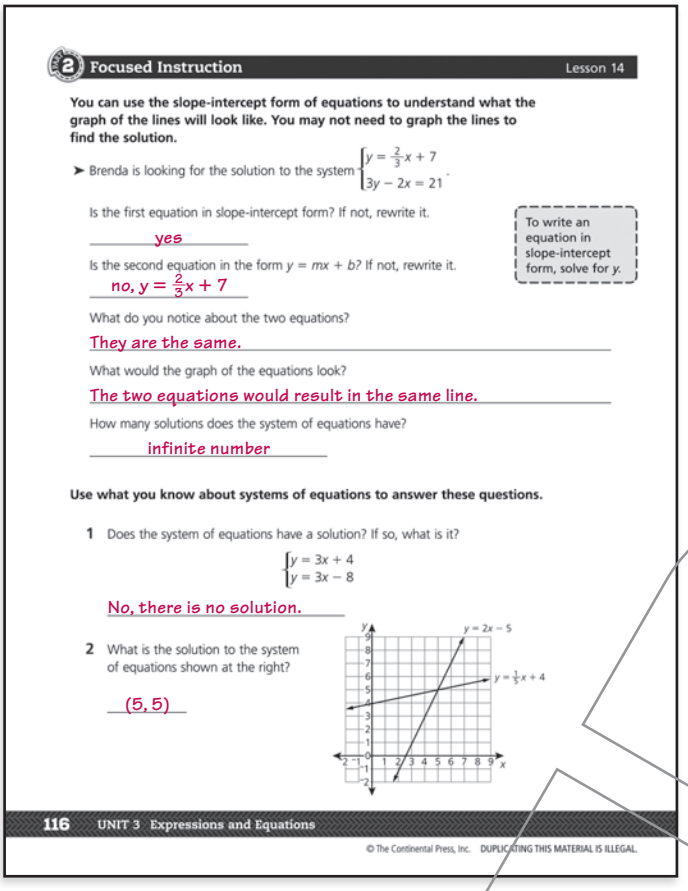

2 Focused Instruction Lesson 14

You can use the slope-intercept form of equations to understand what the graph of the lines will look like. You may not need to graph the lines to find the solution.

➤ Brenda is looking for the solution to the system $\begin{cases} y = \frac{2}{3}x + 7 \\ 3y - 2x = 21 \end{cases}$.

Is the first equation in slope-intercept form? If not, rewrite it.

yes

To write an equation in slope-intercept form, solve for y.

Is the second equation in the form $y = mx + b$? If not, rewrite it.

no, $y = \frac{2}{3}x + 7$

What do you notice about the two equations?

They are the same.

What would the graph of the equations look?

The two equations would result in the same line.

How many solutions does the system of equations have?

infinite number

Use what you know about systems of equations to answer these questions.

1 Does the system of equations have a solution? If so, what is it?

$\begin{cases} y = 3x + 4 \\ y = 3x - 8 \end{cases}$

No, there is no solution.

2 What is the solution to the system of equations shown at the right?

(5, 5)

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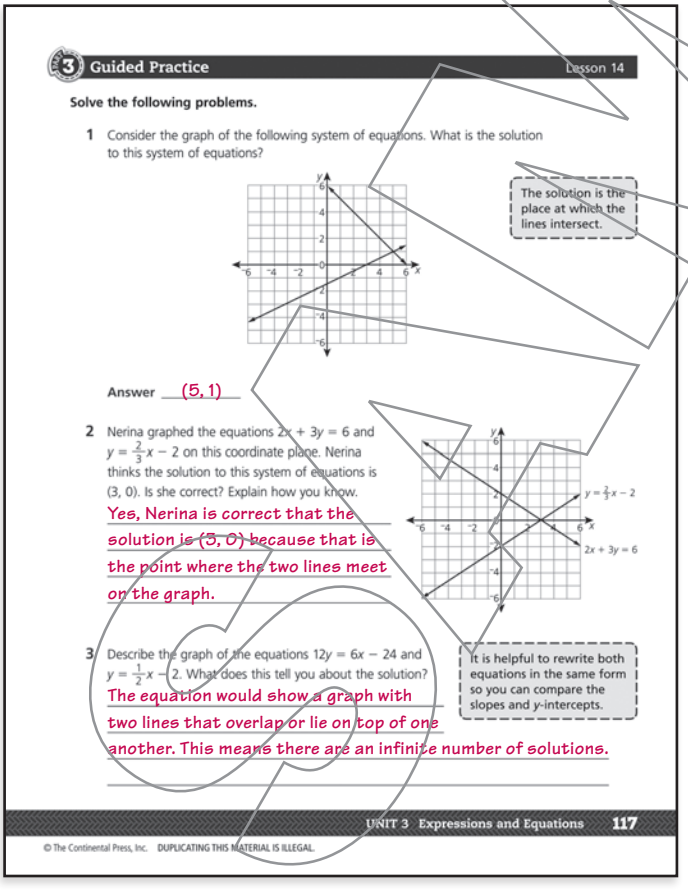
PART 2 Focused Instruction

First, students must find the solution to a system of equations by looking at a graph. They should be able to recognize the point where the two lines intersect, find the ordered pair that describes the point, and understand that this point is the solution to the system of equations.

Next, students will change one equation in a system of equations into slope-intercept form so that they can compare the two equations. By doing so, they should recognize that the equations describe the same line, so there are an infinite number of solutions.

Conclude the Focused Instruction section by having students find the solution, if there is one, to two systems of equations.



3 Guided Practice Lesson 14

Solve the following problems.

1 Consider the graph of the following system of equations. What is the solution to this system of equations?

The solution is the place at which the lines intersect.

Answer (5, 1)

2 Nerina graphed the equations $2x + 3y = 6$ and $y = \frac{2}{3}x - 2$ on this coordinate plane. Nerina thinks the solution to this system of equations is (3, 0). Is she correct? Explain how you know.

Yes, Nerina is correct that the solution is (3, 0) because that is the point where the two lines meet on the graph.

3 Describe the graph of the equations $12y = 6x - 24$ and $y = \frac{1}{2}x - 2$. What does this tell you about the solution?

The equation would show a graph with two lines that overlap or lie on top of one another. This means there are an infinite number of solutions.

It is helpful to rewrite both equations in the same form so you can compare the slopes and y-intercepts.

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PART 3 Guided Practice

Students should complete the Guided Practice section on their own. Offer assistance as needed, pointing out the reminder and hint boxes along the right side of the page.

Connections to Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.

SAMPLE

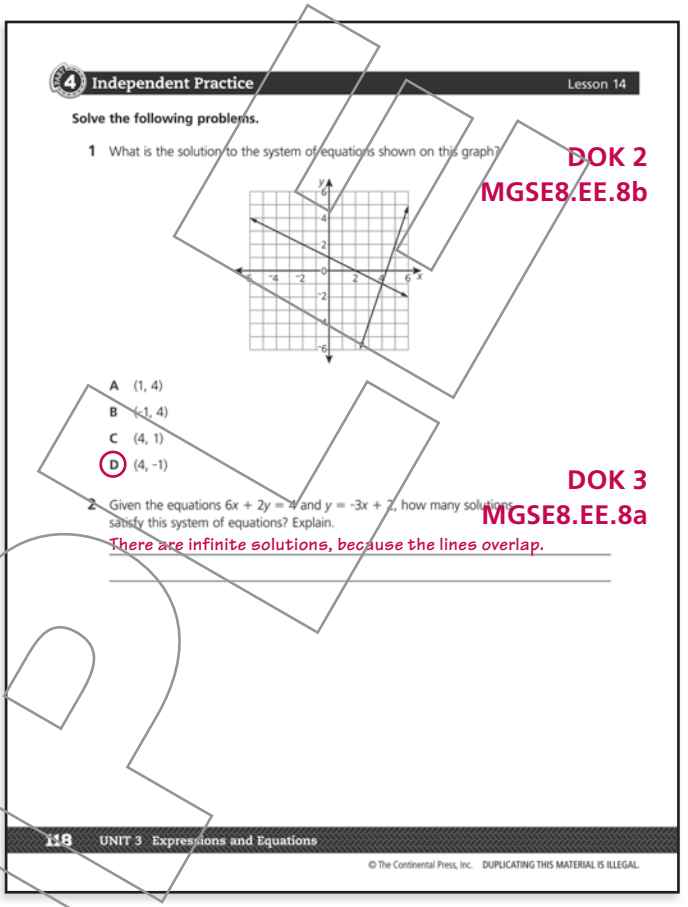
Part 4 Independent Practice Answer Rationales

1 The point where the two lines intersect is the solution to the system of equations. These lines intersect at (4, –1), so choice D is correct. In choice A, the student inverted the *x*- and *y*-values and negated the *y*-value. In choice B, the student inverted the *x*- and *y*-values. In choice C, the student misread the *y*-value.

2 First, change $6x + 2y = 4$ to slope-intercept form in order to compare equations. Solve for *y:* $6x + 2y = 4$; $2y = -6x + 4$; $y = -3x + 2$. Since this equation is the same as the other equation, the lines that represent the equations will also be the same, which means they overlap. A system of equations that results in overlapping lines has infinite solutions because the lines overlap forever in each direction.

3 Choice D is correct because lines *a* and *d* intersect at the point (–1, –3), which means that point is the solution to that system of equations. In choice A, lines *a* and *c* intersect at (–3, –1); the student inverted the *x*- and *y*-values when looking for a solution. In choice B, lines *b* and *c* intersect at (–1, 3); the student chose a solution with the opposite sign for the *y*-value. In choice C, lines *b* and *d* intersect at (3, 1); the student inverted the *x*- and *y*-values and changed their signs.

4 Find the solution to the system of equations visually by locating the point at which the two lines intersect. On this graph, the point is (0, –4). This means that when $x = 0$ and $y = -4$, both equations are true.



4 Independent Practice Lesson 14

Solve the following problems.

1 What is the solution to the system of equations shown on this graph? DOK 2 MGSE8.EE.8b

A (1, 4)
B (–1, 4)
C (4, 1)
D (4, –1)

2 Given the equations $6x + 2y = 4$ and $y = -3x + 2$, how many solutions satisfy this system of equations? Explain. DOK 3 MGSE8.EE.8a

There are infinite solutions, because the lines overlap.

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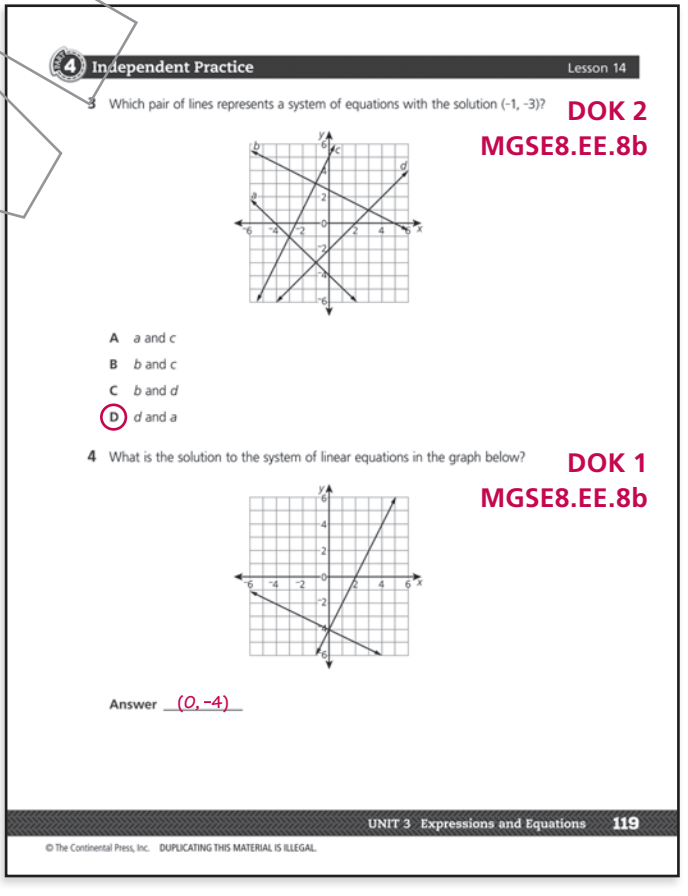

4 Independent Practice Lesson 14

3 Which pair of lines represents a system of equations with the solution (–1, –3)? DOK 2 MGSE8.EE.8b

A *a* and *c*
B *b* and *c*
C *b* and *d*
D *d* and *a*

4 What is the solution to the system of linear equations in the graph below? DOK 1 MGSE8.EE.8b

Answer (0, –4)

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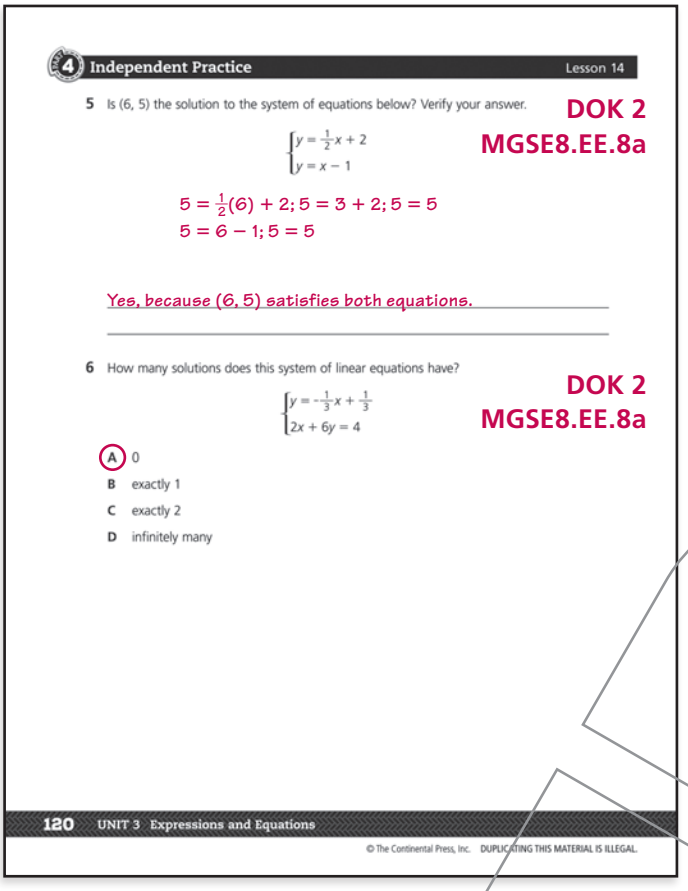

4 Independent Practice Lesson 14

5 Is (6, 5) the solution to the system of equations below? Verify your answer. DOK 2 MGSE8.EE.8a

$$\begin{cases} y = \frac{1}{2}x + 2 \\ y = x - 1 \end{cases}$$

$5 = \frac{1}{2}(6) + 2; 5 = 3 + 2; 5 = 5$
$5 = 6 - 1; 5 = 5$

Yes, because (6, 5) satisfies both equations.

6 How many solutions does this system of linear equations have? DOK 2 MGSE8.EE.8a

$$\begin{cases} y = -\frac{1}{3}x + \frac{1}{3} \\ 2x + 6y = 4 \end{cases}$$

(A) 0
B exactly 1
C exactly 2
D infinitely many

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5 The equations have different slopes and intercepts and so have one solution. To verify the solution, check that the values of x and y satisfy both equations by substituting them into the equations and simplifying. In both equations, when $x = 6$ and $y = 5$, the equations are true, so this point is the solution to the system of equations.

6 To compare the equations, put the second equation in slope-intercept form by solving for y: $2x + 6y = 4$; $6y = -2x + 4$; $y = -\frac{1}{3}x + \frac{2}{3}$. Since this equation has the same slope as the first equation in the system of equations but a different y-intercept, the lines are parallel. There is no solution because they never meet. Choice A is correct. Choice B is incorrect because there would be one solution if the lines only met once. Choice C is incorrect because two linear equations would never meet twice when graphed. Choice D is incorrect because the lines would need to have the same slope and the same y-intercept to have infinite solutions.

Extension Activity

Give each student a blank coordinate plane with axes from −10 to 10. Have each student graph a line on his or her plane and write the equation that describes the line. Prior to doing this activity, prepare pieces of tissue paper the same size as the coordinate planes. Draw various lines on the paper so that when they overlap the students' coordinate planes, they show the graph of a system of equations. Pass out the pieces of tissue paper. Have the students place these lines over their coordinate plane, identify the equation of the new line, and find the solution to the system of equations. Redistribute the pieces of tissue paper and repeat the activity as desired.

SAMPLE