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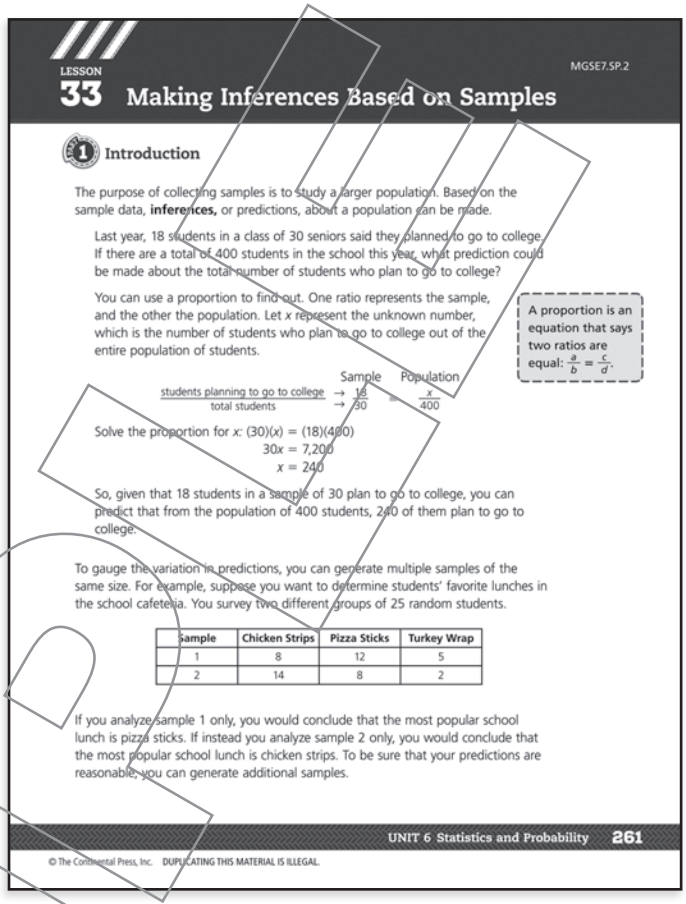
ISBN 978-1-5240-0022-6

Objective

To review making inferences about a population based on sample data

Introduction

Remind students that the purpose of sampling is to provide insight into a population. Discuss inferences as predictions based on data. Work through the example on the page to demonstrate how to use a proportion consisting of a ratio for the sample and a second ratio for the population to find a missing value that represents the prediction. Stress that the greater the number of samples, the more reasonable the prediction is likely to be.



LESSON 33 Making Inferences Based on Samples

MGSE7.SP.2

1 Introduction

The purpose of collecting samples is to study a larger population. Based on the sample data, **inferences,** or predictions, about a population can be made.

Last year, 18 students in a class of 30 seniors said they planned to go to college. If there are a total of 400 students in the school this year, what prediction could be made about the total number of students who plan to go to college?

You can use a proportion to find out. One ratio represents the sample, and the other the population. Let x represent the unknown number, which is the number of students who plan to go to college out of the entire population of students.

A proportion is an equation that says two ratios are equal: $\frac{a}{b} = \frac{c}{d}$.

$$\frac{\text{students planning to go to college}}{\text{total students}} \quad \begin{matrix}\text{Sample} \\ \rightarrow \frac{18}{30} \end{matrix} = \begin{matrix}\text{Population} \\ \frac{x}{400}\end{matrix}$$

Solve the proportion for x: $(30)(x) = (18)(400)$

$30x = 7{,}200$

$x = 240$

So, given that 18 students in a sample of 30 plan to go to college, you can predict that from the population of 400 students, 240 of them plan to go to college.

To gauge the variation in predictions, you can generate multiple samples of the same size. For example, suppose you want to determine students' favorite lunches in the school cafeteria. You survey two different groups of 25 random students.

Sample	Chicken Strips	Pizza Sticks	Turkey Wrap
1	8	12	5
2	14	8	2

If you analyze sample 1 only, you would conclude that the most popular school lunch is pizza sticks. If instead you analyze sample 2 only, you would conclude that the most popular school lunch is chicken strips. To be sure that your predictions are reasonable, you can generate additional samples.

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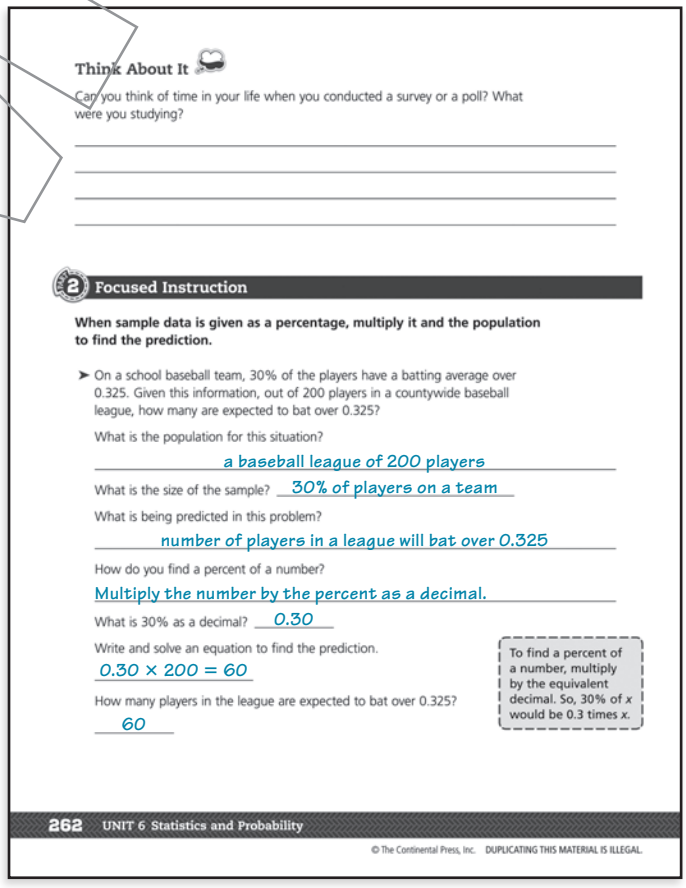
Think About It

Students should recall an instance in which they conducted a survey or poll, for example, a classroom survey of their breakfast or pastime preferences.



Think About It

Can you think of time in your life when you conducted a survey or a poll? What were you studying?

2 Focused Instruction

When sample data is given as a percentage, multiply it and the population to find the prediction.

➤ On a school baseball team, 30% of the players have a batting average over 0.325. Given this information, out of 200 players in a countywide baseball league, how many are expected to bat over 0.325?

What is the population for this situation?

a baseball league of 200 players

What is the size of the sample? 30% of players on a team

What is being predicted in this problem?

number of players in a league will bat over 0.325

How do you find a percent of a number?

Multiply the number by the percent as a decimal.

What is 30% as a decimal? 0.30

Write and solve an equation to find the prediction.

0.30 × 200 = 60

To find a percent of a number, multiply by the equivalent decimal. So, 30% of x would be 0.3 times x.

How many players in the league are expected to bat over 0.325?

60

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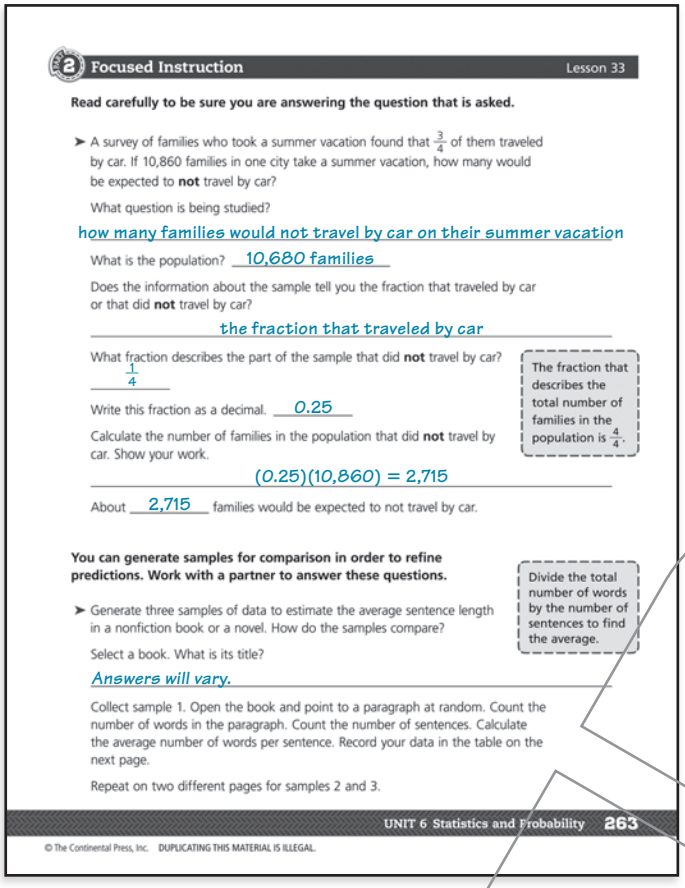
Georgia Standard of Excellence

MGSE7.SP.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

Vocabulary

inference: an educated guess or prediction based on statistics



Focused Instruction Lesson 33

Read carefully to be sure you are answering the question that is asked.

➤ A survey of families who took a summer vacation found that $\frac{3}{4}$ of them traveled by car. If 10,860 families in one city take a summer vacation, how many would be expected to **not** travel by car?

What question is being studied?

how many families would not travel by car on their summer vacation

What is the population? 10,680 families

Does the information about the sample tell you the fraction that traveled by car or that did **not** travel by car?

the fraction that traveled by car

What fraction describes the part of the sample that did **not** travel by car? $\frac{1}{4}$

The fraction that describes the total number of families in the population is $\frac{4}{4}$.

Write this fraction as a decimal. 0.25

Calculate the number of families in the population that did **not** travel by car. Show your work.

(0.25)(10,860) = 2,715

About 2,715 families would be expected to not travel by car.

You can generate samples for comparison in order to refine predictions. Work with a partner to answer these questions.

Divide the total number of words by the number of sentences to find the average.

➤ Generate three samples of data to estimate the average sentence length in a nonfiction book or a novel. How do the samples compare?

Select a book. What is its title?

Answers will vary.

Collect sample 1. Open the book and point to a paragraph at random. Count the number of words in the paragraph. Count the number of sentences. Calculate the average number of words per sentence. Record your data in the table on the next page.

Repeat on two different pages for samples 2 and 3.

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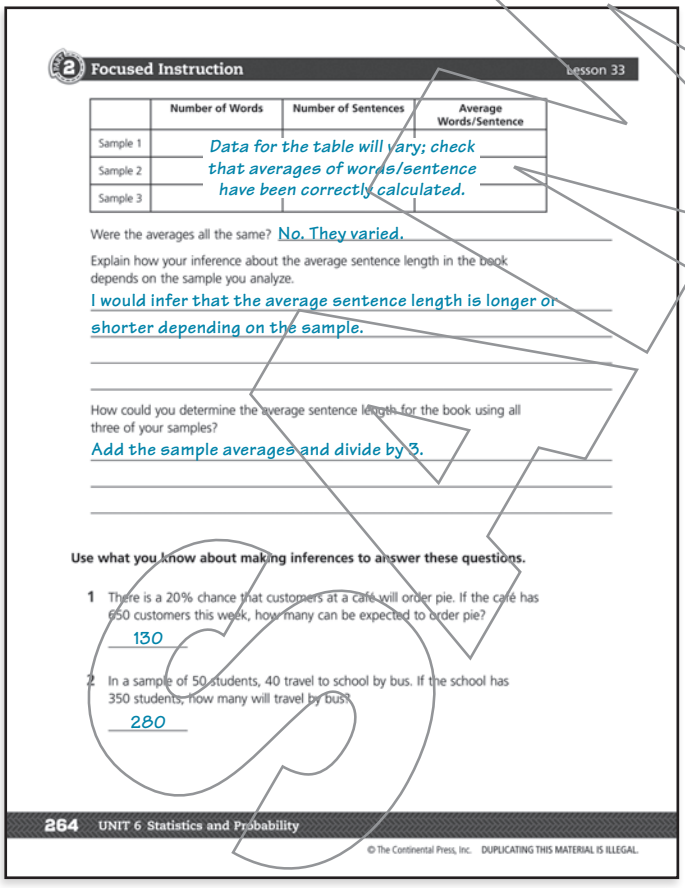

Focused Instruction Lesson 33

	Number of Words	Number of Sentences	Average Words/Sentence
Sample 1			
Sample 2			
Sample 3			

Data for the table will vary; check that averages of words/sentence have been correctly calculated.

Were the averages all the same? No. They varied.

Explain how your inference about the average sentence length in the book depends on the sample you analyze.

I would infer that the average sentence length is longer or shorter depending on the sample.

How could you determine the average sentence length for the book using all three of your samples?

Add the sample averages and divide by 3.

Use what you know about making inferences to answer these questions.

1 There is a 20% chance that customers at a café will order pie. If the café has 650 customers this week, how many can be expected to order pie?

130

2 In a sample of 50 students, 40 travel to school by bus. If the school has 350 students, how many will travel by bus?

280

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PART 2 Focused Instruction

First, students make a prediction for a population given only a percentage of a sample, rather than the numbers. Changing the percent to a decimal, they write and solve an equation to find the predicted number. Next, students make a prediction using a fraction.

Then, students are guided in generating samples, gathering data, and drawing their own conclusions and inferences. Students count the words and sentences in three randomly selected paragraphs of a book of their choice, and find the average number of words per sentence in each sample, recording their findings in a table. They use this data to draw conclusions.

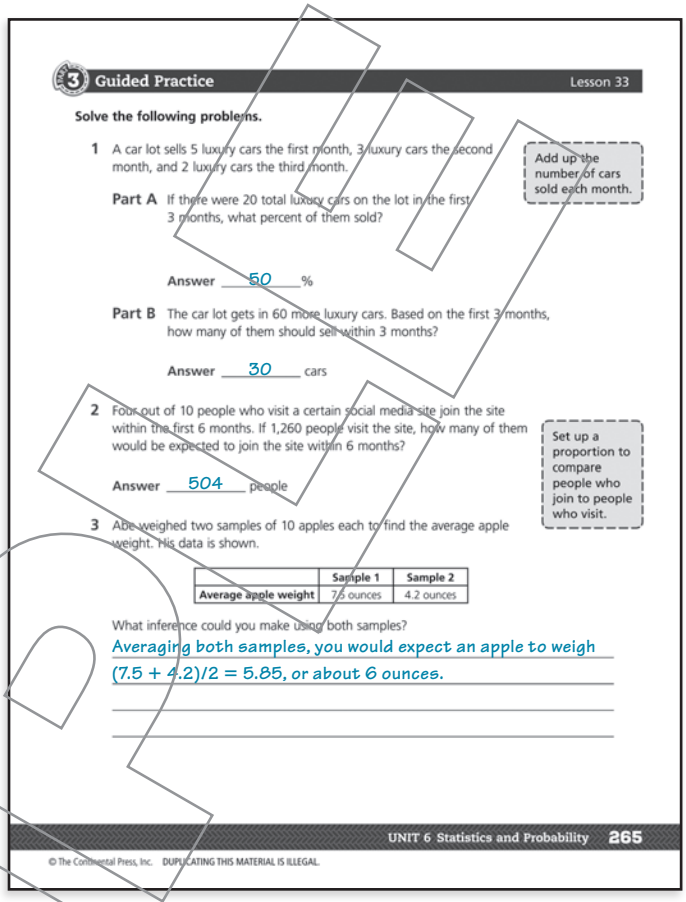
Conclude the Focused Instruction section by having students answer two questions involving making inferences.

Connections to Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.

PART 3 Guided Practice

Students should complete the Guided Practice section on their own. Offer assistance as needed, pointing out the reminder and hint boxes along the right side of the page.



3 Guided Practice Lesson 33

Solve the following problems.

1 A car lot sells 5 luxury cars the first month, 3 luxury cars the second month, and 2 luxury cars the third month.

Add up the number of cars sold each month.

Part A If there were 20 total luxury cars on the lot in the first 3 months, what percent of them sold?

Answer 50 %

Part B The car lot gets in 60 more luxury cars. Based on the first 3 months, how many of them should sell within 3 months?

Answer 30 cars

2 Four out of 10 people who visit a certain social media site join the site within the first 6 months. If 1,260 people visit the site, how many of them would be expected to join the site within 6 months?

Set up a proportion to compare people who join to people who visit.

Answer 504 people

3 Abe weighed two samples of 10 apples each to find the average apple weight. His data is shown.

	Sample 1	Sample 2
Average apple weight	7.5 ounces	4.2 ounces

What inference could you make using both samples?

Averaging both samples, you would expect an apple to weigh (7.5 + 4.2)/2 = 5.85, or about 6 ounces.

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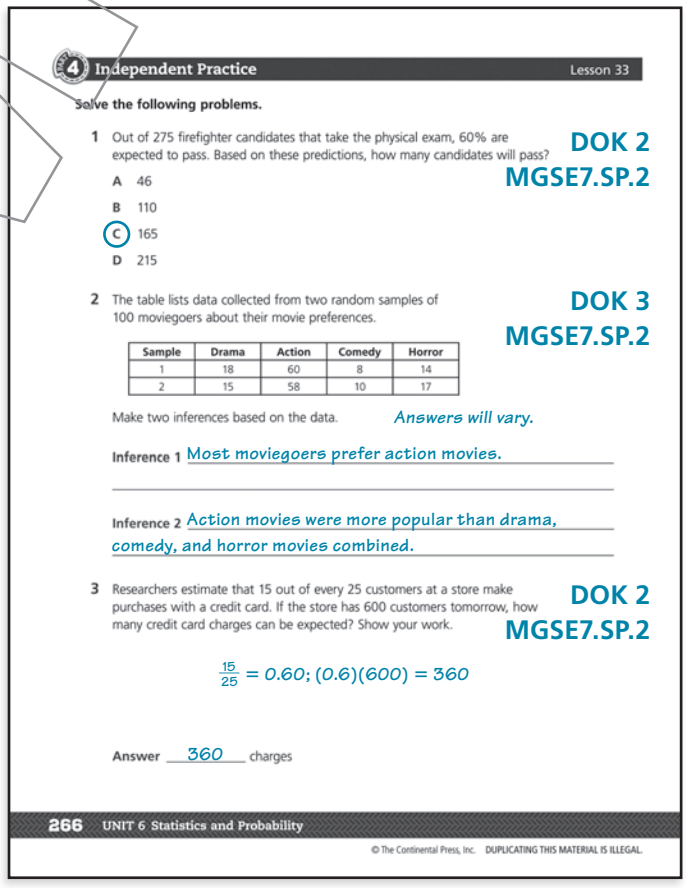
PART 4 Independent Practice Answer Rationales

1 The proportion that represents the situation is $\frac{60}{100} = \frac{n}{275}$, which simplifies to the equation $n = 0.6(275)$. To find n, the number of candidates that are expected to pass, multiply the total number of candidates by 0.6, the decimal form of 60%: $n =$ 165 candidates. Choice C is correct. Choice A is incorrect because it shows 275 divided by 6. Choice B is incorrect because it results from subtracting 0.6 from 1.0 and uses the difference 0.4 to multiply. Choice D is incorrect because it simply subtracts 60 from 275 to get 215.

2 Both samples show similar data. Out of 200 total values, drama is 16.5%, action is 59%, comedy is 9%, and horror is 15.5%. Inferences may include that action movies are the most popular, comedy movies are the least popular, and action movies are more popular than all other movie types combined.

3 Set up the ratios representing the researchers' estimates of customers using credit cards, $\frac{15}{25}$, and the store's ratio of customers using credit cards for purchases, $\frac{x}{600}$. Cross multiply: $25(x) = 15(600) =$ 9,000; $x = \frac{9{,}000}{25} = 360$. Alternatively, rewrite $\frac{15}{25}$ as 0.6 and multiply: $x = 0.6(600) = 360$ customers.



4 Independent Practice Lesson 33

Solve the following problems.

1 Out of 275 firefighter candidates that take the physical exam, 60% are expected to pass. Based on these predictions, how many candidates will pass?

DOK 2 MGSE7.SP.2

A 46
B 110
C 165
D 215

2 The table lists data collected from two random samples of 100 moviegoers about their movie preferences.

DOK 3 MGSE7.SP.2

Sample	Drama	Action	Comedy	Horror
1	18	60	8	14
2	15	58	10	17

Make two inferences based on the data. Answers will vary.

Inference 1 Most moviegoers prefer action movies.

Inference 2 Action movies were more popular than drama, comedy, and horror movies combined.

3 Researchers estimate that 15 out of every 25 customers at a store make purchases with a credit card. If the store has 600 customers tomorrow, how many credit card charges can be expected? Show your work.

DOK 2 MGSE7.SP.2

$\frac{15}{25} = 0.60$; $(0.6)(600) = 360$

Answer 360 charges

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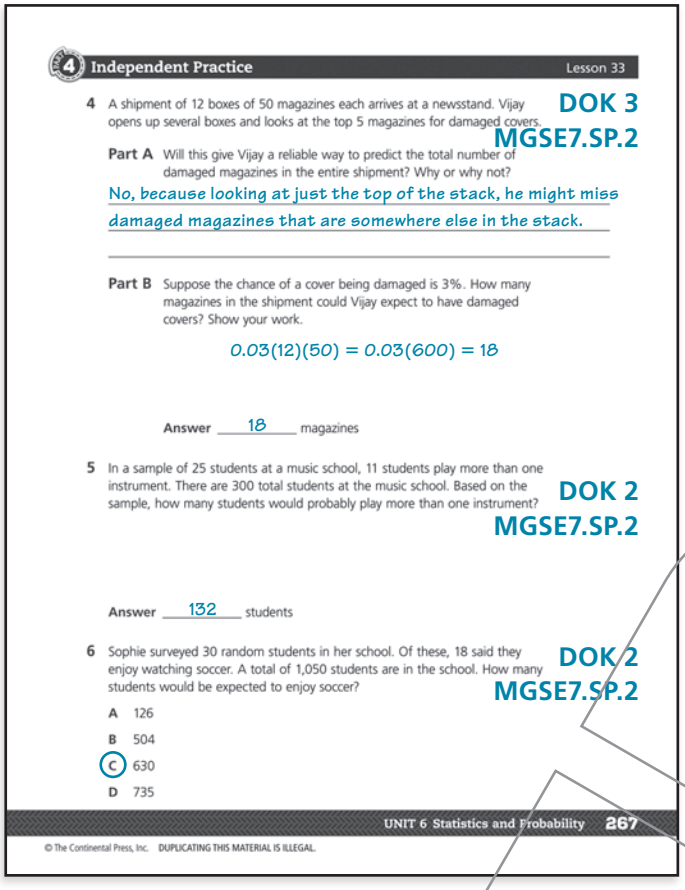

4 Independent Practice Lesson 33

4 A shipment of 12 boxes of 50 magazines each arrives at a newsstand. Vijay opens up several boxes and looks at the top 5 magazines for damaged covers. DOK 3 MGSE7.SP.2

Part A Will this give Vijay a reliable way to predict the total number of damaged magazines in the entire shipment? Why or why not?

No, because looking at just the top of the stack, he might miss damaged magazines that are somewhere else in the stack.

Part B Suppose the chance of a cover being damaged is 3%. How many magazines in the shipment could Vijay expect to have damaged covers? Show your work.

0.03(12)(50) = 0.03(600) = 18

Answer 18 magazines

5 In a sample of 25 students at a music school, 11 students play more than one instrument. There are 300 total students at the music school. Based on the sample, how many students would probably play more than one instrument? DOK 2 MGSE7.SP.2

Answer 132 students

6 Sophie surveyed 30 random students in her school. Of these, 18 said they enjoy watching soccer. A total of 1,050 students are in the school. How many students would be expected to enjoy soccer? DOK 2 MGSE7.SP.2

A 126
B 504
C 630
D 735

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4 PART A Looking at just the top magazines excludes other areas that could be damaged. The sample is not representative of the population.

PART B Convert 3% to 0.03 and multiply it by the total number of magazines: $0.03(12)(50) = 0.03(600) = 18$. Vijay could expect 18 damaged covers.

5 The ratios representing the number of students who play more than one instrument in the sample is $\frac{11}{25}$. The ratio of total students who play more than one instrument is $\frac{x}{300}$. Set up a proportion and solve for x: $25(x) = 11(300) = 3{,}300$; $x = \frac{3{,}300}{25} = 132$ students.

6 To find the number of students expected to enjoy soccer, set up a proportion with ratios for the sample and the population: $\frac{18}{30} = \frac{x}{1{,}050}$. Solve for x: $30x = 18{,}900$; $x = \frac{18{,}900}{30} = 630$ students. Choice C is correct. Choice A is incorrect; it uses the difference between 18 and 30, expressed as the decimal 0.12, to multiply the total number of students in the school. Choice B is incorrect; it adds 18 to 30 and expresses the sum as a decimal, 0.48, which multiplies the total number of students in the school. Choice D is incorrect; it subtracts 30 from 100 and expresses the difference as a decimal, 0.70, which multiplies the total number of students in the school.

Extension Activity

Divide the class into two groups for two samples. Ask each student to count the letters in his or her first and last names. Draw a chart like the following for each group and have students record their numbers in the appropriate cells.

	Less than 5 letters	5 to 9 letters	More than 9 letters
First name	3, 4, 4, 4	5, 5, 6, 8, 9	10, 10, 12
Last name	2, 3, 4	5, 7, 7, 9	10, 12, 12, 15, 18

Then have students draw inferences from the two samples and find a percent for each category. Then give them the number of students in seventh grade and in their school, and ask them to predict the number of students with names of various lengths. Save the data for use with later extension activities.