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10 Ratio Reasoning with Measurement Conversions

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Objective

To use rates to convert measurements

Part 1 Introduction

Students should be familiar with some equivalent measurements, such as 12 inches equal 1 foot. Discuss how unit rates can be used to find equivalent measurements, using the skills they learned earlier with rates and unit rates.

Think About It

Students should recognize that converting from a larger unit to a smaller unit will result in a larger number because it will take more of the smaller unit to make the same measurement. In the same way, converting from a smaller unit to a larger unit will result in a smaller number because it will take fewer of the larger unit to make the measurement.

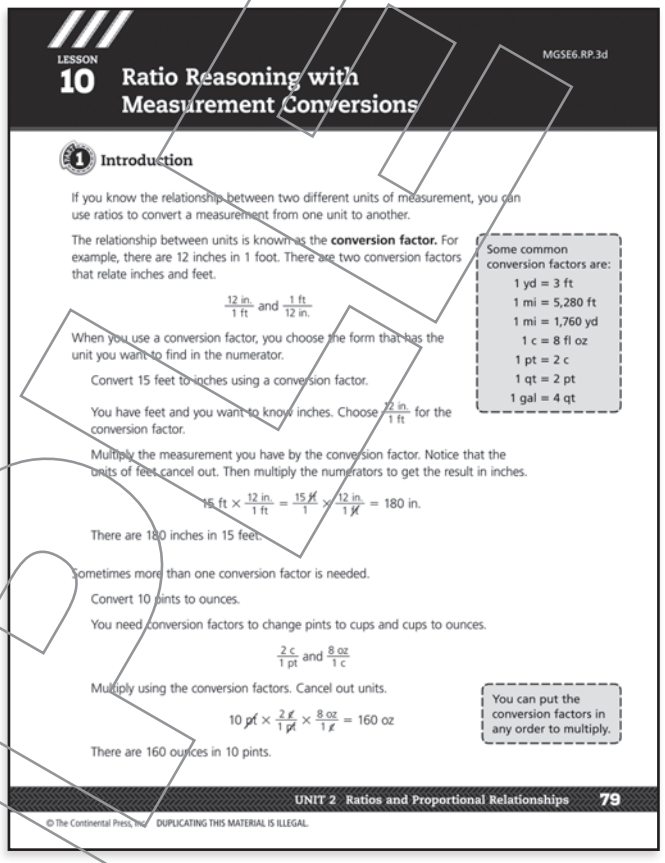
Georgia Standard of Excellence

MGSE6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems utilizing strategies such as tables of equivalent ratios, tape diagrams (bar models), double number line diagrams, and/or equations.

3d. Given a conversion factor, use ratio reasoning to convert measurement units within one system of measurement and between two systems of measurements (customary and metric); manipulate and transform units appropriately when multiplying or dividing quantities.

Vocabulary

conversion factor: a ratio of equal measure used to change a rate with one set of measurements to another



LESSON 10 Ratio Reasoning with Measurement Conversions

MGSE6.RP.3d

1 Introduction

If you know the relationship between two different units of measurement, you can use ratios to convert a measurement from one unit to another.

The relationship between units is known as the **conversion factor.** For example, there are 12 inches in 1 foot. There are two conversion factors that relate inches and feet.

$\frac{12 \text{ in.}}{1 \text{ ft}}$ and $\frac{1 \text{ ft}}{12 \text{ in.}}$

When you use a conversion factor, you choose the form that has the unit you want to find in the numerator.

Convert 15 feet to inches using a conversion factor.

You have feet and you want to know inches. Choose $\frac{12 \text{ in.}}{1 \text{ ft}}$ for the conversion factor.

Multiply the measurement you have by the conversion factor. Notice that the units of feet cancel out. Then multiply the numerators to get the result in inches.

$$15 \text{ ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} = \frac{15 \text{ ft}}{1} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 180 \text{ in.}$$

There are 180 inches in 15 feet.

Sometimes more than one conversion factor is needed.

Convert 10 pints to ounces.

You need conversion factors to change pints to cups and cups to ounces.

$\frac{2 \text{ c}}{1 \text{ pt}}$ and $\frac{8 \text{ oz}}{1 \text{ c}}$

Multiply using the conversion factors. Cancel out units.

$$10 \text{ pt} \times \frac{2 \text{ c}}{1 \text{ pt}} \times \frac{8 \text{ oz}}{1 \text{ c}} = 160 \text{ oz}$$

There are 160 ounces in 10 pints.

Some common conversion factors are:
1 yd = 3 ft
1 mi = 5,280 ft
1 mi = 1,760 yd
1 c = 8 fl oz
1 pt = 2 c
1 qt = 2 pt
1 gal = 4 qt

You can put the conversion factors in any order to multiply.

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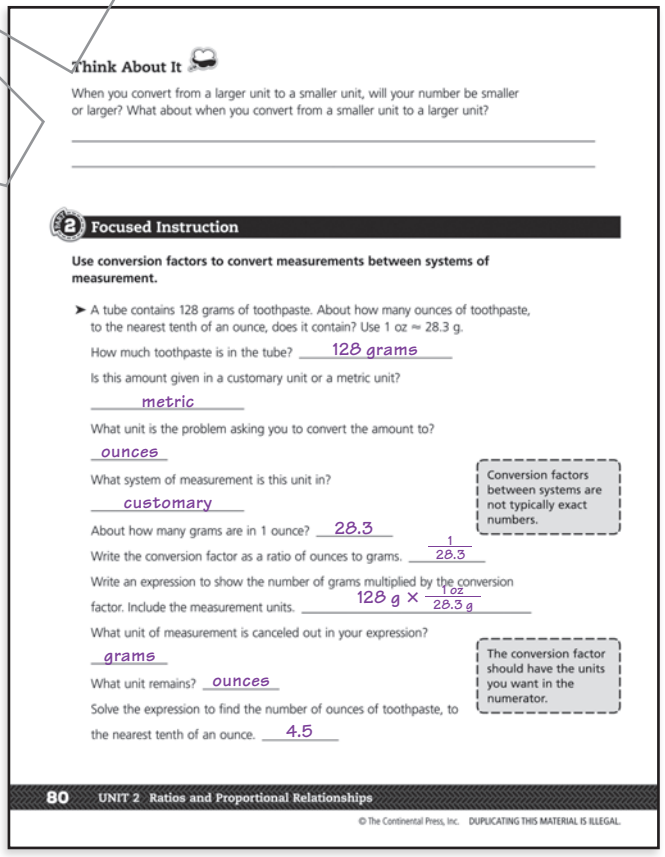

Think About It

When you convert from a larger unit to a smaller unit, will your number be smaller or larger? What about when you convert from a smaller unit to a larger unit?

2 Focused Instruction

Use conversion factors to convert measurements between systems of measurement.

➤ A tube contains 128 grams of toothpaste. About how many ounces of toothpaste, to the nearest tenth of an ounce, does it contain? Use 1 oz ≈ 28.3 g.

How much toothpaste is in the tube? 128 grams

Is this amount given in a customary unit or a metric unit? metric

What unit is the problem asking you to convert the amount to? ounces

What system of measurement is this unit in? customary

About how many grams are in 1 ounce? 28.3

Write the conversion factor as a ratio of ounces to grams. $\frac{1}{28.3}$

Write an expression to show the number of grams multiplied by the conversion factor. Include the measurement units. $128 \text{ g} \times \frac{1 \text{ oz}}{28.3 \text{ g}}$

What unit of measurement is canceled out in your expression? grams

What unit remains? ounces

Solve the expression to find the number of ounces of toothpaste, to the nearest tenth of an ounce. 4.5

Conversion factors between systems are not typically exact numbers.

The conversion factor should have the units you want in the numerator.

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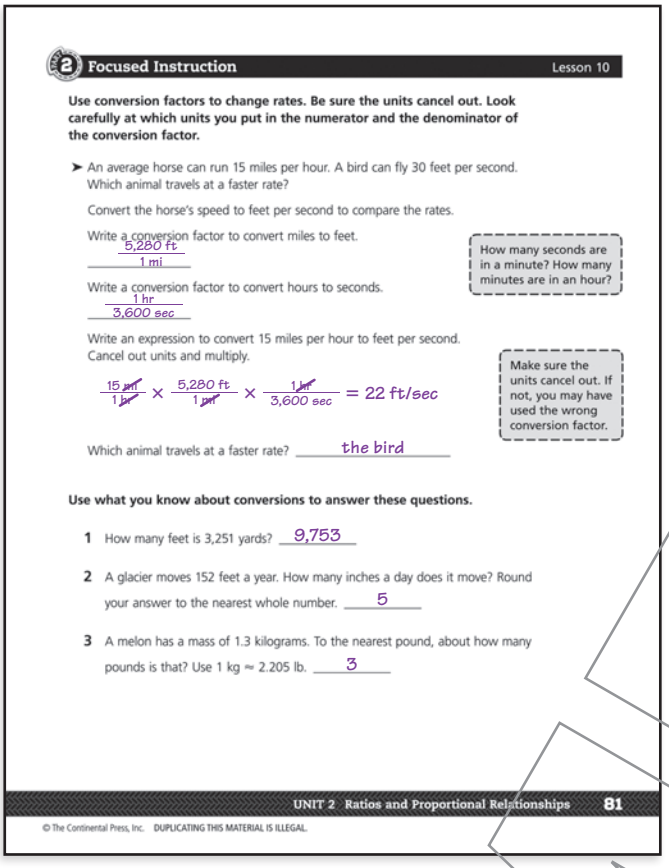

Focused Instruction Lesson 10

Use conversion factors to change rates. Be sure the units cancel out. Look carefully at which units you put in the numerator and the denominator of the conversion factor.

➤ An average horse can run 15 miles per hour. A bird can fly 30 feet per second. Which animal travels at a faster rate?

Convert the horse's speed to feet per second to compare the rates.

Write a conversion factor to convert miles to feet.

$\frac{5{,}280 \text{ ft}}{1 \text{ mi}}$

How many seconds are in a minute? How many minutes are in an hour?

Write a conversion factor to convert hours to seconds.

$\frac{1 \text{ hr}}{3{,}600 \text{ sec}}$

Write an expression to convert 15 miles per hour to feet per second. Cancel out units and multiply.

$\frac{15 \text{ mi}}{1 \text{ hr}} \times \frac{5{,}280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{3{,}600 \text{ sec}} = 22 \text{ ft/sec}$

Make sure the units cancel out. If not, you may have used the wrong conversion factor.

Which animal travels at a faster rate? the bird

Use what you know about conversions to answer these questions.

1 How many feet is 3,251 yards? 9,753

2 A glacier moves 152 feet a year. How many inches a day does it move? Round your answer to the nearest whole number. 5

3 A melon has a mass of 1.3 kilograms. To the nearest pound, about how many pounds is that? Use 1 kg ≈ 2.205 lb. 3

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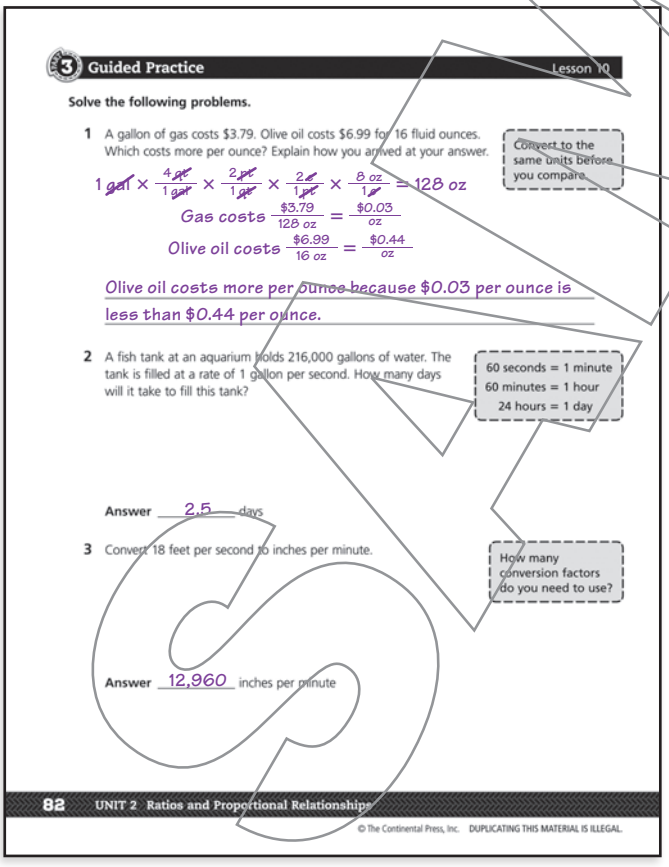

Guided Practice Lesson 10

Solve the following problems.

1 A gallon of gas costs $3.79. Olive oil costs $6.99 for 16 fluid ounces. Which costs more per ounce? Explain how you arrived at your answer.

Convert to the same units before you compare.

$1 \text{ gal} \times \frac{4 \text{ qt}}{1 \text{ gal}} \times \frac{2 \text{ pt}}{1 \text{ qt}} \times \frac{2 \text{ c}}{1 \text{ pt}} \times \frac{8 \text{ oz}}{1 \text{ c}} = 128 \text{ oz}$

Gas costs $\frac{\$3.79}{128 \text{ oz}} = \frac{\$0.03}{\text{oz}}$

Olive oil costs $\frac{\$6.99}{16 \text{ oz}} = \frac{\$0.44}{\text{oz}}$

Olive oil costs more per ounce because $0.03 per ounce is less than $0.44 per ounce.

2 A fish tank at an aquarium holds 216,000 gallons of water. The tank is filled at a rate of 1 gallon per second. How many days will it take to fill this tank?

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day

Answer 2.5 days

3 Convert 18 feet per second to inches per minute.

How many conversion factors do you need to use?

Answer 12,960 inches per minute

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PART 2 Focused Instruction

First, students will convert grams to ounces. They will work through questions that help them find the conversion factor and complete the conversion.

Next, students will use conversion factors to change rates. In the problem, they must change miles per hour to feet per second in order to compare two speeds. Students must set up an expression that uses multiple conversion factors in order to find the desired outcome.

Conclude the Focused Instruction section by having students answer two questions involving measurement conversions.

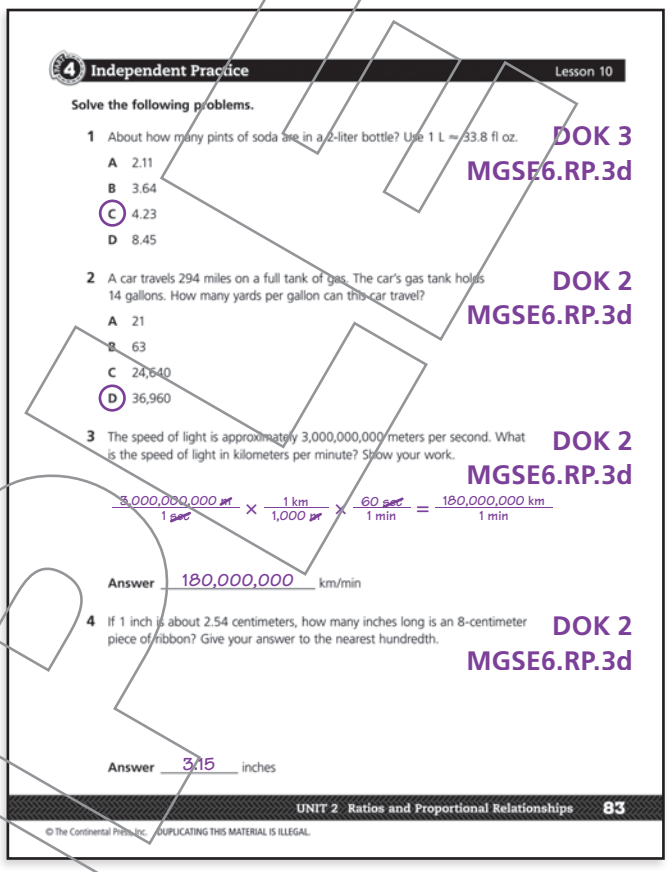
PART 3 Guided Practice

Students should complete the Guided Practice section on their own. Offer assistance as needed, pointing out the reminder and hint boxes along the right side of the page.

Connections to Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Model with mathematics
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

PART 4 Independent Practice Answer Rationales



4 Independent Practice Lesson 10

Solve the following problems.

1 About how many pints of soda are in a 2-liter bottle? Use 1 L ≈ 33.8 fl oz. DOK 3 MGSE6.RP.3d

A 2.11
B 3.64
C 4.23
D 8.45

2 A car travels 294 miles on a full tank of gas. The car's gas tank holds 14 gallons. How many yards per gallon can this car travel? DOK 2 MGSE6.RP.3d

A 21
B 63
C 24,640
D 36,960

3 The speed of light is approximately 3,000,000,000 meters per second. What is the speed of light in kilometers per minute? Show your work. DOK 2 MGSE6.RP.3d

$\frac{3{,}000{,}000{,}000 \text{ m}}{1 \text{ sec}} \times \frac{1 \text{ km}}{1{,}000 \text{ m}} \times \frac{60 \text{ sec}}{1 \text{ min}} = \frac{180{,}000{,}000 \text{ km}}{1 \text{ min}}$

Answer 180,000,000 km/min

4 If 1 inch is about 2.54 centimeters, how many inches long is an 8-centimeter piece of ribbon? Give your answer to the nearest hundredth. DOK 2 MGSE6.RP.3d

Answer 3.15 inches

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1 The given conversion factor is for liters and fluid ounces. The problem asks for the number of pints. Use the given conversion factor as well as the conversion factor for fluid ounces and pints. There are 16 fluid ounces in 1 pint. Set up an expression: $2 \text{ L} \times \frac{33.8 \text{ fl oz}}{1 \text{ L}} \times \frac{1 \text{ pt}}{16 \text{ fl oz}}$. When solving, the liters and the fluid ounces will cancel out because they both appear in the numerator and the denominator. The pints remain, so the answer is in pints: $2 \cancel{\text{L}} \times \frac{33.8 \cancel{\text{fl oz}}}{1 \cancel{\text{L}}} \times \frac{1 \text{ pt}}{16 \cancel{\text{fl oz}}} = \frac{2 \times 33.8}{16} = 4.225$. The choices show numbers rounded to the nearest hundredth: 4.225 rounds to 4.23, which is choice C.

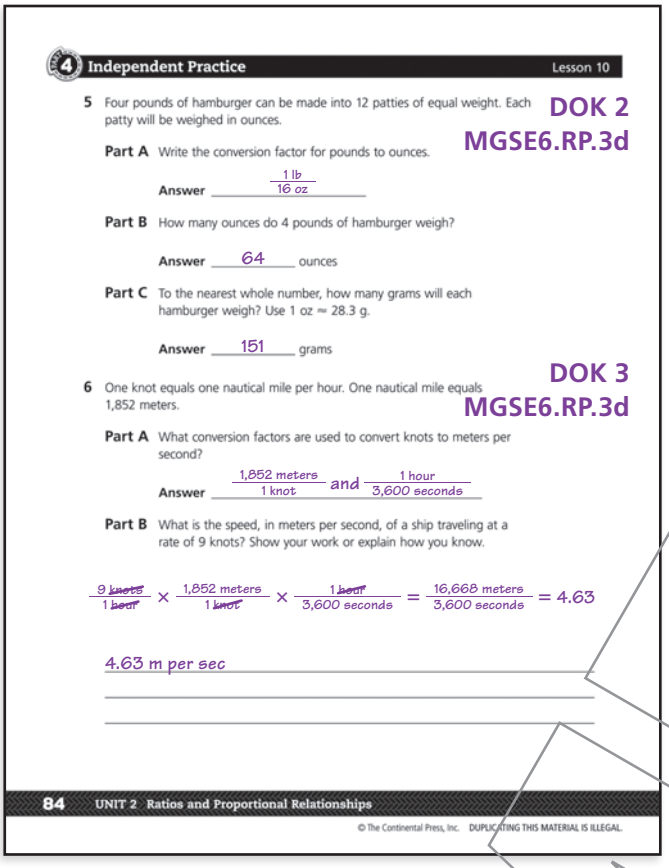
2 Use the conversion factor for miles to yards. You do not need to convert the gallons. Set up the expression and multiply: $\frac{294 \text{ mi}}{14 \text{ gal}} \times \frac{1{,}760 \text{ yd}}{1 \text{ mi}} = \frac{517{,}440 \text{ yd}}{14 \text{ gal}} = \frac{36{,}960 \text{ yd}}{1 \text{ gal}}$. Choice D is correct.

3 Use the conversion factors for meters to kilometers $\frac{1 \text{ km}}{1{,}000 \text{ m}}$ and seconds to minutes $\frac{60 \text{ sec}}{1 \text{ min}}$. Be sure to cancel out units as needed: $\frac{3{,}000{,}000{,}000 \text{ m}}{1 \text{ sec}}$ is equivalent to $\frac{18{,}000{,}000 \text{ km}}{1 \text{ min}}$.

4 The conversion factor to change centimeters to inches is $\frac{1 \text{ in.}}{2.54 \text{ cm}}$. Write an expression to change 8 centimeters to inches: $8 \text{ cm} \times \frac{1 \text{ in.}}{2.54 \text{ cm}} = 3.1496\ldots$. Rounded to the nearest hundredth, the length is 3.15 inches.

Extension Activity

Research or have students research average rates for a variety of things, such as speed of a cheetah and water flowing over a certain waterfall. You may also have students brainstorm rates from their own lives, such as text messages sent per day and hours of soccer practice per week. Have students convert the rates using conversion factors. For example, if the speed of a cheetah is found in yards per minute, convert it to feet per minute, inches per hour, etc.



4 Independent Practice — Lesson 10

5 Four pounds of hamburger can be made into 12 patties of equal weight. Each patty will be weighed in ounces. DOK 2 MGSE6.RP.3d

Part A Write the conversion factor for pounds to ounces.

Answer $\frac{1 \text{ lb}}{16 \text{ oz}}$

Part B How many ounces do 4 pounds of hamburger weigh?

Answer 64 ounces

Part C To the nearest whole number, how many grams will each hamburger weigh? Use 1 oz ≈ 28.3 g.

Answer 151 grams

6 One knot equals one nautical mile per hour. One nautical mile equals 1,852 meters. DOK 3 MGSE6.RP.3d

Part A What conversion factors are used to convert knots to meters per second?

Answer $\frac{1{,}852 \text{ meters}}{1 \text{ knot}}$ and $\frac{1 \text{ hour}}{3{,}600 \text{ seconds}}$

Part B What is the speed, in meters per second, of a ship traveling at a rate of 9 knots? Show your work or explain how you know.

$\frac{9 \text{ knots}}{1 \text{ hour}} \times \frac{1{,}852 \text{ meters}}{1 \text{ knot}} \times \frac{1 \text{ hour}}{3{,}600 \text{ seconds}} = \frac{16{,}668 \text{ meters}}{3{,}600 \text{ seconds}} = 4.63$

4.63 m per sec

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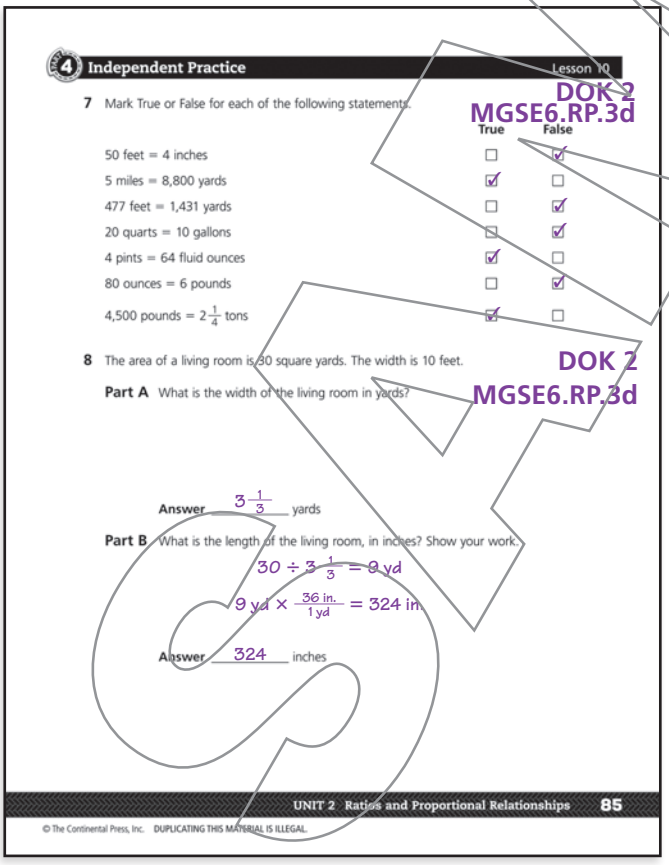

4 Independent Practice — Lesson 10

7 Mark True or False for each of the following statements. DOK 2 MGSE6.RP.3d

	True	False
50 feet = 4 inches	☐	☑
5 miles = 8,800 yards	☑	☐
477 feet = 1,431 yards	☐	☑
20 quarts = 10 gallons	☐	☑
4 pints = 64 fluid ounces	☑	☐
80 ounces = 6 pounds	☐	☑
4,500 pounds = $2\frac{1}{4}$ tons	☑	☐

8 The area of a living room is 30 square yards. The width is 10 feet. DOK 2 MGSE6.RP.3d

Part A What is the width of the living room in yards?

Answer $3\frac{1}{3}$ yards

Part B What is the length of the living room, in inches? Show your work.

$30 \div 3\frac{1}{3} = 9$ yd

$9 \text{ yd} \times \frac{36 \text{ in.}}{1 \text{ yd}} = 324$ in.

Answer 324 inches

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5 PART A There are 16 ounces in 1 pound, so the conversion factor is written as $\frac{1 \text{ lb}}{16 \text{ oz}}$.

PART B Use the conversion factor and multiply: $4 \text{ lb} \times \frac{16 \text{ oz}}{1 \text{ lb}} = 64$ oz. So 4 pounds of hamburger weigh 64 ounces.

PART C First, find the number of ounces for each hamburger. Divide the total weight by 12: $64 \div 12 = 5.33\ldots$ ounces. Then use the conversion factor $\frac{28.3 \text{ g}}{1 \text{ oz}}$. Since you want the answer in grams, be sure grams is in the numerator: $5.33\ldots \times \frac{28.3}{1} \approx 150.93\ldots \approx 151$. Each hamburger is about 151 grams.

6 PART A Since 1 knot is equal to 1,852 meters, the conversion factors to use are $\frac{1{,}852 \text{ meters}}{1 \text{ knot}}$ and $\frac{1 \text{ hour}}{3{,}600 \text{ seconds}}$.

PART B Set up an expression using the conversion factors. Cancel out a unit when it appears in both the numerator and the denominator. The units that remain should be meters in the numerator and seconds in the denominator so that the result is in meters per second.

7 Use the correct conversion factors to check each choice. The first, third, fourth, and sixth statements are incorrect: $50 \text{ ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 600$ in., not 4 in.; $477 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} = 159$ yd, not 1,431 yd; $20 \text{ qt} \times \frac{1 \text{ gal}}{4 \text{ qt}} = 5$ gal, not 10 gal; and $80 \text{ oz} \times \frac{1 \text{ lb}}{16 \text{ oz}} = 5$ lb, not 6 lb. The second, fifth, and seventh statements are true: $5 \text{ mi} \times \frac{1{,}760 \text{ yd}}{1 \text{ mi}} = 8{,}800$ yd; $4 \text{ pt} \times \frac{2 \text{ c}}{1 \text{ pt}} \times \frac{8 \text{ oz}}{1 \text{ c}} = 64$ oz; and $4{,}500 \text{ lb} \times \frac{1 \text{ T}}{2{,}000 \text{ lb}} = 2\frac{1}{4}$ T.

8 PART A Convert feet to yards: $10 \text{ ft} \times \frac{1 \text{ yd}}{3 \text{ ft}} = 3\frac{1}{3}$ yd. So the width of the living room is $3\frac{1}{3}$ yards.

PART B The formula for the area of a rectangle is *Area = length × width.* To find the length, divide the area by the width: 30 yards $\div 3\frac{1}{3}$ yards = 9 yards. The length of the living room is 9 yards. To change this to inches, use two conversion factors, $\frac{3 \text{ ft}}{1 \text{ yd}}$ and $\frac{12 \text{ in.}}{1 \text{ ft}}$, or one conversion factor, $\frac{36 \text{ in.}}{1 \text{ yd}}$:

$9 \text{ yd} \times \frac{36 \text{ in.}}{1 \text{ yd}} = 324$ in.